Neutrinos in dense matter: beyond modified Urca

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Washington University in St. Louis

M. Alford, A. Haber, Z. Zhang, arXiv:2406.13717

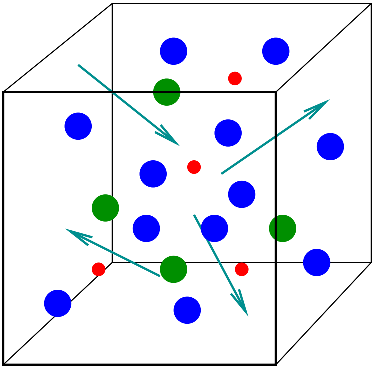


Outline

1. Importance of Urca processes
2. The direct Urca + modified Urca approximation
3. Problems with modified Urca
4. A better way: the Nucleon Width Approximation

Nuclear matter fluid

Generic fluid element



neutrons: dominant constituent

protons: small fraction

electrons: maintaining local neutrality

neutrinos: *not always thermally equilibrated!*

Fluid is described by 3-4 parameters:

$\boxed{n_B} = n_\mathsf{n} + n_\mathsf{p}$ baryon density

$\boxed{T}$ temperature

$\boxed{x_p} = n_\mathsf{p}/n_B$ proton fraction

$\left(\boxed{x_L} = n_L/n_B \quad \text{lepton fraction}\right)$

[if neutrinos are thermally equilibrated]

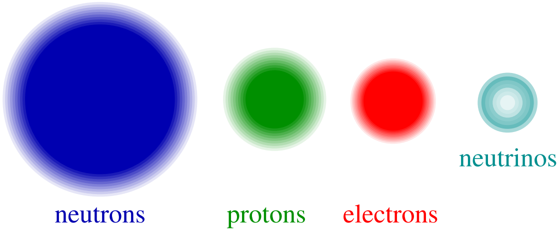
Nuclear matter: degenerate fermions

Neutrons, protons, and electrons are always thermally equilibrated into Fermi seas because they feel strong or electromagnetic interactions.

Neutrinos can have a long mfp and may not be thermally equilibrated

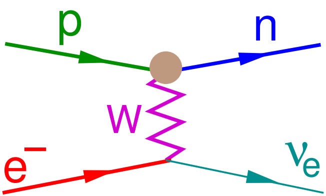
$$n \longleftrightarrow p \quad e^- \quad \bar{\nu}$$

$$\nu \quad n \longleftrightarrow p \quad e^-$$



At $T \ll E_F$, beta equilibration is dominated by Urca processes involving modes near the Fermi surfaces

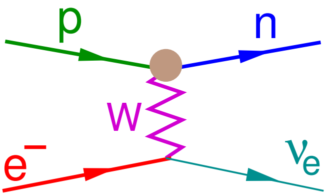
The importance of Urca



Typical Urca processes
in dense nuclear matter:

$$\mathsf{n} \leftrightarrow \mathsf{p} + \mathsf{e}^- + \bar{\nu}_e$$
$$\mathsf{p} + \mathsf{e}^- \leftrightarrow \mathsf{n} + \nu_e$$

The importance of Urca



Typical Urca processes in dense nuclear matter:

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$
$$p + e^- \leftrightarrow n + \nu_e$$

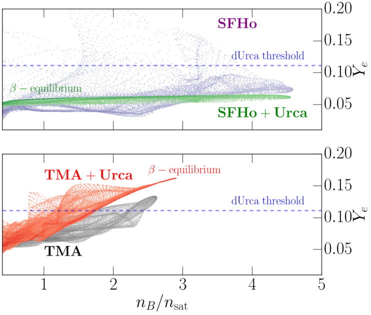
Urca processes are the driver of:

- Relaxation of the proton fraction $\Rightarrow$ bulk viscosity, damping
- Neutrino absorption opacity (mean free path)
- If ν_e mfp is short: Relaxation of neutrino fraction, shear viscosity
- If ν_e mfp is long: Neutrino emissivity

Relevant in:

- supernovas (neutrino opacity, deleptonization)
- individual neutron stars (cooling)
- neutron star mergers (neutrino opacity, isospin relaxation)

Examples in mergers

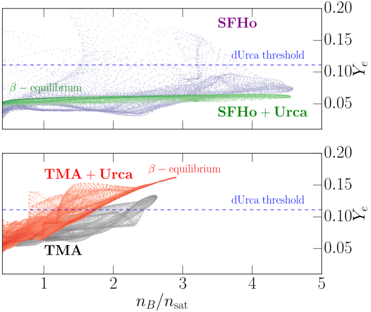


Density distribution of electron fraction Y_e measured 5 ms after merger

Note difference between the distributions with and without Urca processes.

Most et. al., `arXiv:2207.00442` (ApJ Lett)

Examples in mergers



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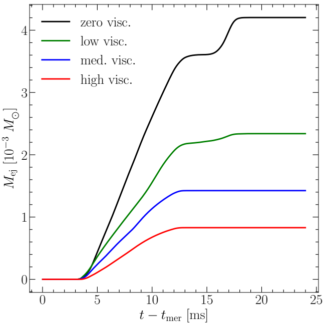
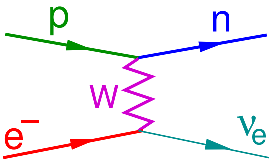


Ejected mass depends on bulk viscosity

Chabanov & Rezzolla `arXiv:2311.13027`

Direct Urca rate

"Direct Urca" means we only include strong interactions via mean-field effects: nucleon effective mass and energy shift.
It is then easy to calculate the rate as a function of (n_B, T, x_p).



$$\Gamma^{\text{dUrca}}_{p\,e^-\to n\,\nu} \sim$$

$$\int d^3k_n d^3k_p d^3k_e d^3k_\nu \underbrace{f_p\, f_e\,(1-f_n)\,(1-f_\nu)}_{\text{particle distributions}}$$

$$\underbrace{\delta^4(k_n - k_p - k_e - k_\nu)}_{\text{energy \& mom cons}} \underbrace{\left|\mathcal{M}_{\text{dU}}(\vec{k}_n, \vec{k}_p, \vec{k}_e, \vec{k}_\nu)\right|^2}_{\text{Matrix element}}$$

Reduces to 4D integral, or low-T analytic expression.

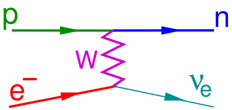
Note that the neutron, proton, and electron occupation distributions are thermally equilibrated Fermi-Dirac functions, but the neutrinos may have some non-thermal occupation distribution.

Direct and Modified Urca

In general we expect there will be strong-interaction corrections to the simple dUrca diagram. Standard approach includes one correction:

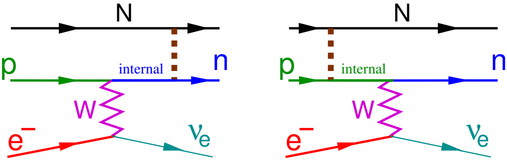
Standard approach: Direct Urca rate + *approx* Modified Urca rate

Direct Urca



4-dimensional integral

Modified Urca



11-dimensional integral

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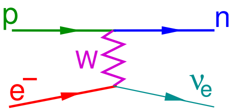

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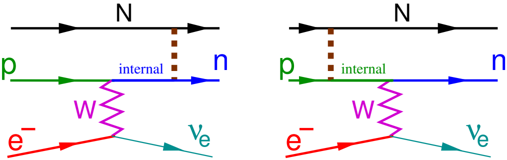
Modified Urca



11-dimensional integral

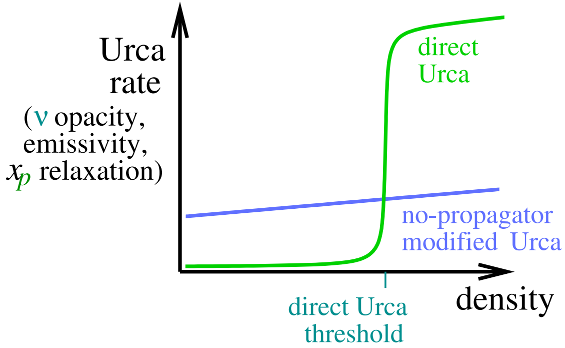
☹ mURca needs severe approximations to make it evaluate-able, e.g. Fermi surface approx, and neglect internal propagator!

☹ mUrca is difficult to improve, e.g. if we include internal propagator then the rate diverges when internal particles go on shell

☹ mUrca is difficult to generalize, e.g. to non-zero **magnetic field**

Why do people think they need Modified Urca?

There are situations where strong interaction corrections are essential.
E.g. in cool neutrino-transparent matter



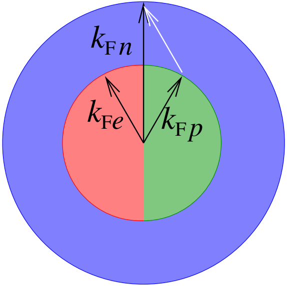
dUrca threshold varies by EoS: for some there is *no* dUrca!

Why is there a dUrca threshold?

$$p \;\; e^- \leftrightarrow n \;\; \nu_e$$

High proton fraction

Direct Urca open



$\vec{k}_n = \vec{k}_p + \vec{k}_e$ is possible

because $k_{F,n} < k_{F,p} + k_{F,e}$

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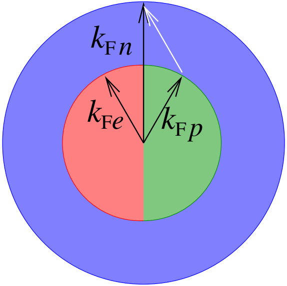

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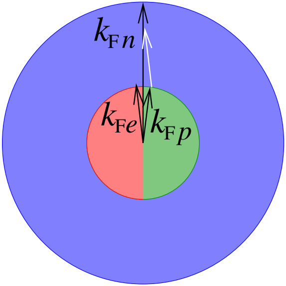

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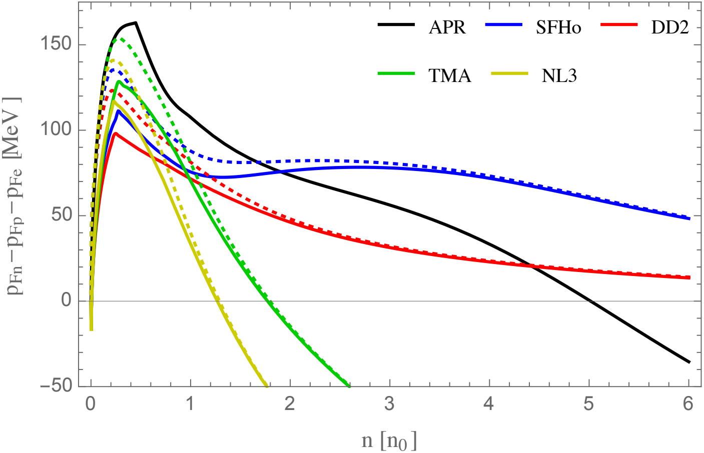
Threshold softened by:

- Thermal blurring of Fermi surfaces
- Nucleon width (grows as T^2)

Direct Urca threshold

Some examples of the direct Urca kinematic constraint

Direct Urca is unsuppressed when $k_{F,n} - k_{F,p} - k_{F,e} < 0$



Some EoSes by have no direct Urca at any density:

need strong interaction corrections: modified Urca?

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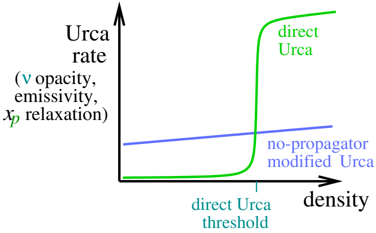

$\longrightarrow$

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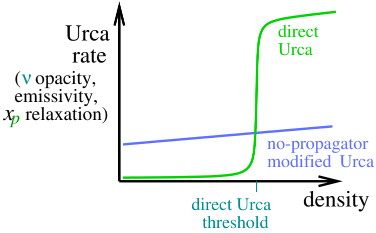
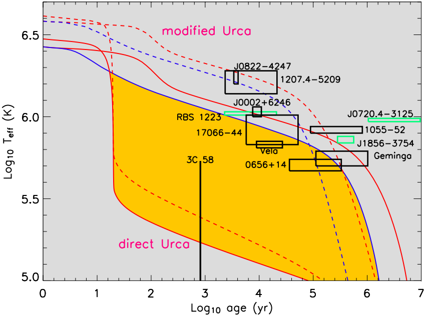


$\rightarrow$

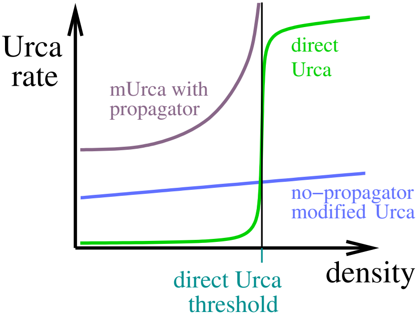


Lattimer and Prakash
`arXiv:astro-ph/0405262`

But no-propagator mUrca is a bad estimate
Can we improve it?

“Improved” modified Urca is unusable!

Try including the propagator for the internal nucleon in modified Urca



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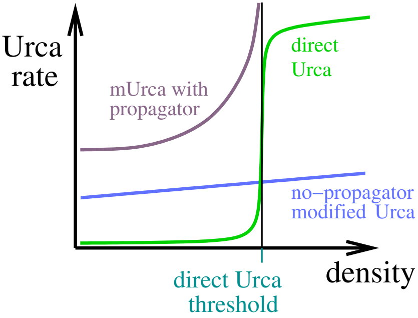
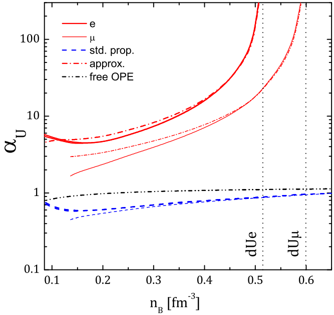




Shternin, Baldo, Haensel, arXiv:1807.06569

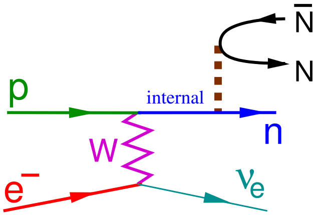
☹ No-propagator mUrca is wrong by a factor of 10, even far below dUrca threshold

☹ Including the propagator ⇒ mUrca diverges at the dUrca threshold!

Is there a better way to handle strong-interaction corrections?

What is modified Urca trying to do?

We can rewrite modified Urca:



The strong interaction lets a nucleon radiate a nucleon particle-hole pair with 3-momentum but little energy.

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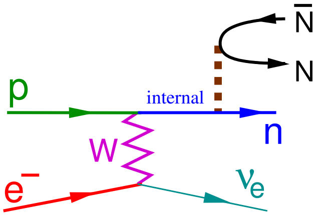

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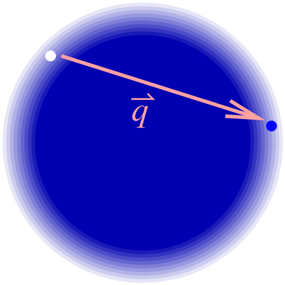

In a dense medium this is easy because there are lots of available states with very low energy cost.

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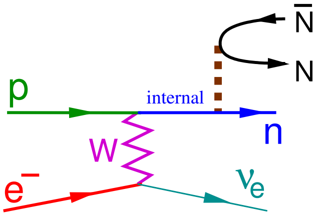

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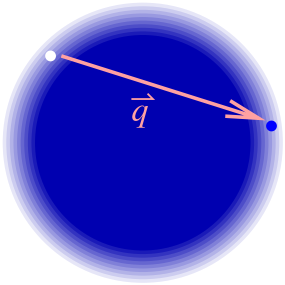


In a dense medium this is easy because there are lots of available states with very low energy cost.

Modified Urca is trying to remind us that in a dense medium,

the nucleon is *unstable*.

the nucleon has a non-zero *width*

the nucleon's energy has an *imaginary part*

A nucleon width is better than mUrca

If we allow nucleons to have widths due to their strong interactions with the medium, we can implement the physics behind modified Urca without any unphysical divergences.

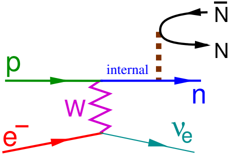
An unstable particle's propagator takes the form

$$= G_n(E_n, \vec{k}_n) = \frac{(4\times 4 \text{ Dirac matrix})}{{E_n}^2 - |\vec{k}_n|^2 - (M_n - i\Gamma_n/2)^2}$$

Thanks to the width Γ_n, even when the internal particle is "on shell",

$$E^2 = |\vec{k}|^2 + M^2$$

the propagator doesn't diverge.



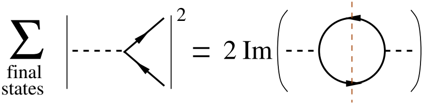
How do we implement particle widths in the Urca rate?

Rates for unstable particles: Cutkosky rules

$$\text{Rate} \propto \text{Im}\begin{pmatrix}\text{initial state} \\ \text{self-energy}\end{pmatrix}$$

Optical theorem for quantum fields

E.g.:



$$\left(\begin{array}{l} \text{Where from? Scattering matrix } S_{fi} = I_{fi} + iT_{fi} \\ \underbrace{S^\dagger S = I}_{\text{Unitarity}} \;\Rightarrow\; \sum_f T^\dagger_{if} T_{fi} + i(T_{ii} - T^\dagger_{ii}) + I = I \\ \qquad\qquad\;\Rightarrow \sum_f T^\dagger_{if} T_{fi} = 2\,\text{Im}(T_{ii}) \end{array}\right)$$

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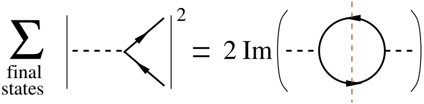

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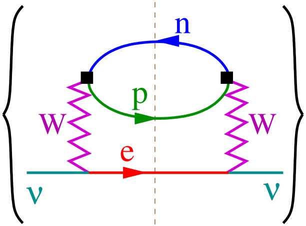
Apply to dUrca: $\frac{\mathrm{d}n_\nu}{\mathrm{d}t} = \mathrm{Im}$

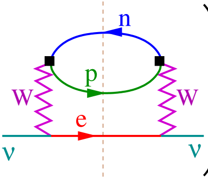


See, e.g., D. Voskresensky [astro-ph/0101514],
A. Sedrakian, A. Dieperink [astro-ph/0002228]

Nucleon Width Approximation

Direct Urca only

$$\frac{dn_\nu}{dt} = \mathrm{Im}\left\{ \right.$$



$$\left. \right\}$$

where nucleon has real mass

$$= \frac{1}{\not{k} - M_n}$$

(and same for proton)

and we neglect vertex corrections

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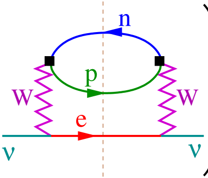

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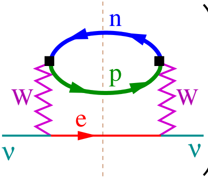

$$\left. \right\}$$

where nucleon has complex mass

$$= \frac{1}{\not{k} - M_n - i\Gamma_n/2}$$

(and same for proton)

and we neglect vertex corrections,
which include cross terms in mUrca

Implementing a nucleon width

Nucleon with mass M and width Γ:

$$= G(k, M + i\Gamma/2) = \frac{1}{\not{k} - M - i\Gamma/2}$$

How do we evaluate a Feynman diagram with such propagators?

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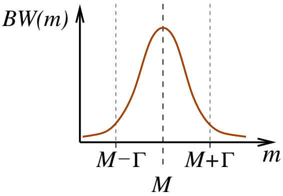
As a spectral representation: Smear the (real) mass using a Breit-Wigner function:

$$G(k, M+i\Gamma/2) = \int_{-\infty}^{\infty} dm \;\; G(k, m) \;\; \underbrace{\mathsf{BW}(m, M, \Gamma)}$$



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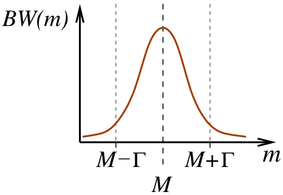
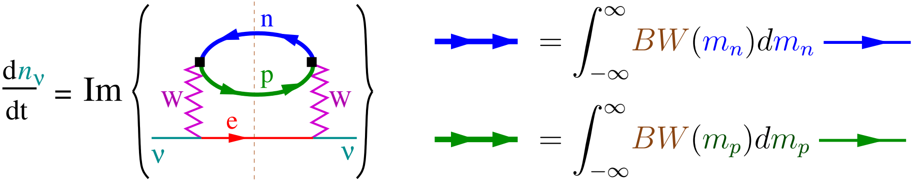
$$\mathsf{BW}(m, M, \Gamma) = \frac{1}{\pi} \frac{\Gamma/2}{(m - M)^2 + \Gamma^2/4}$$



Final assembly

In the Nucleon Width Approx the Urca rate depends linearly on the neutron propagator and on the proton propagator



So the NWA Urca rate is just the dUrca rate, with the nucleon masses smeared out via Breit-Wigner distributions.

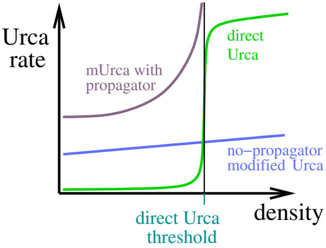
$$\Gamma^{\text{NWA}}(M_n, M_p, \Gamma_n, \Gamma_p) = \int_{-\infty}^{\infty} dm_n dm_p\, \Gamma^{\text{dUrca}}(m_n, m_p)\; \text{BW}(m_n, M_n, \Gamma_n)\, \text{BW}(m_p, M_p, \Gamma_p)$$

Use a model of the strong interaction between nucleons to estimate appropriate values for the nucleon widths $\Gamma_n(n_B, T)$ and $\Gamma_p(n_B, T)$.

NWA versus modified Urca

Recall the divergence problem when we include internal propagator in mUrca

Does NWA fix this?



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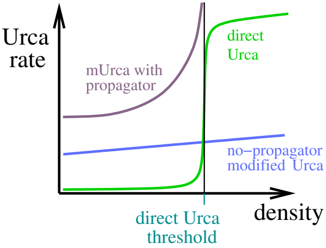
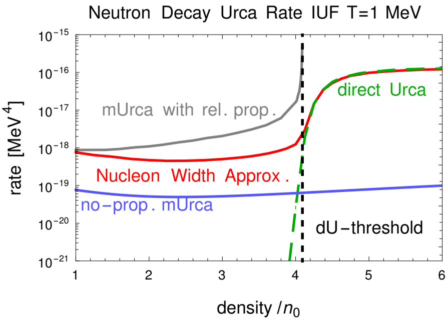


Does NWA fix this? Yes!



Using $\Gamma_{n,p} = T^2/T_0$ with $T_0 = 5\,\text{MeV}$

(Sedrakian & Dieperink, arXiv:astro-ph/0002228)

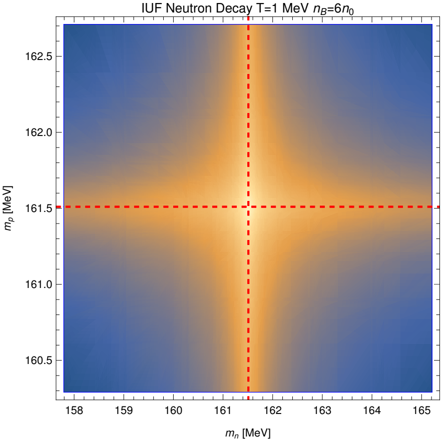
Advantages of the Nucleon Width Approx

$$\Gamma^{\text{NWA}} = \int_{-\infty}^{\infty} dm_n dm_p \, \Gamma^{\text{dUrca}}(m_n, m_p) \, \text{BW}_n(m_n) \text{BW}_p(m_p)$$

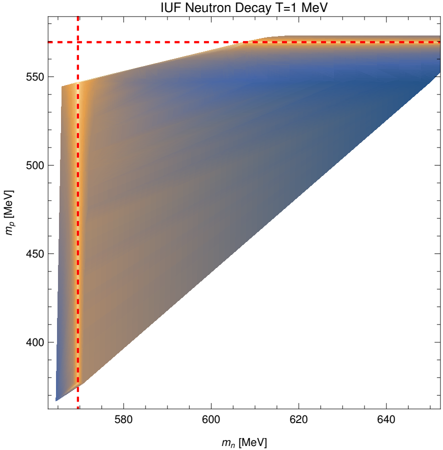
- Agrees with best previous calculations above and far below dUrca threshold.
- Distills strong interaction effects into width parameters Γ_n and Γ_p
- Easily generalized, e.g. to high temperatures, nonzero magnetic field.
- Straightforward to evaluate:
  $T \lesssim 1$ MeV: 2D integral of analytic dUrca expression
  $T \gtrsim 1$ MeV: 6D integral (2 masses + 4 momenta) of full dUrca
- Can be systematically improved

NWA above and below threshold

$$\Gamma^{\text{NWA}} = \int_{-\infty}^{\infty} dm_n dm_p \, \Gamma^{\text{dUrca}}(m_n, m_p) \, \text{BW}_n(m_n) \text{BW}_p(m_p)$$



dUrca is kinematically allowed for the physical nucleon masses



dUrca is kinematically forbidden for the physical nucleon masses

Conclusions

Urca processes $p\ e^- \leftrightarrow n\ \nu_e$ are important for neutrino dynamics (mean free path, emissivity) and isospin equilibration

$$\Gamma^{\text{NWA}} = \int_{-\infty}^{\infty} dm_n dm_p\, \Gamma^{\text{dUrca}}(m_n, m_p)\, \text{BW}_n(m_n) \text{BW}_p(m_p)$$

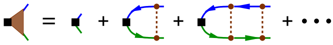
Nucleon Width Approx distills strong interaction corrections into imaginary masses (widths) for neutron and proton.

Better than traditional direct + modified Urca approx:

- Avoids unphysical divergence of modified Urca
- Easy to explore different strong interaction models: just calculate widths Γ_n and Γ_p
- Easy to generalize, e.g. to high T, nonzero magnetic field.
- Easy to evaluate: numerical integral of a positive peaked integrand
- Can be systematically improved (vertex corrections, improved self-energy)

Next steps

- Do a consistent NWA calculation using chiral effective theory for EoS, dispersion relations, and widths.
- Revisit existing calculations that use modified Urca, e.g. neutron star cooling, neutrino opacities, isospin relaxation, etc
- Explore generalizations, e.g. magnetic fields
- Include vertex corrections, starting with RPA iterated strong interaction

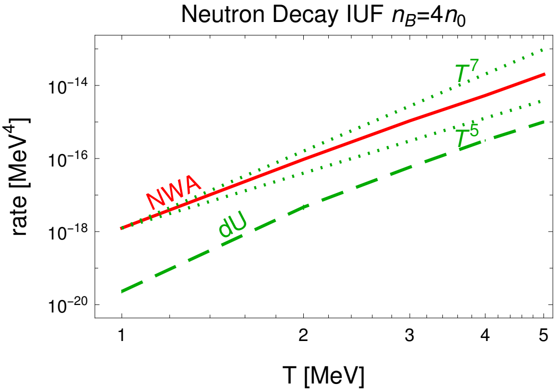


- More complete nucleon self-energy:
  — allow momentum and/or energy dependence
  — different Dirac structures e.g. γ_0, $q^i\gamma_i$
- Superfluid nucleons: use Nambu-Gorkov propagators + widths
- Non-relativistic formulation: A. Sedrakian `arXiv:2406.16183`

Extra slides

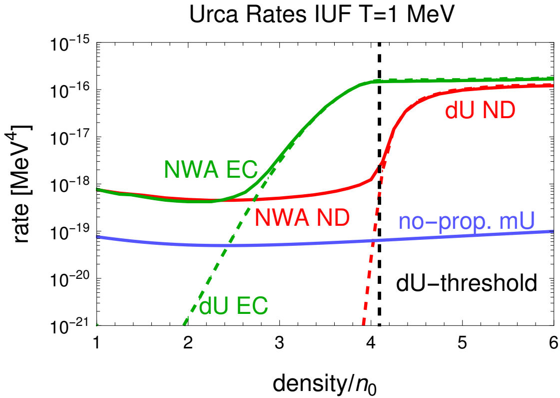
Temperature dependence of NWA rate

dUrca rate $\sim T^5$,
mUrca rate $\sim T^7$;
near threshold, NWA is between the two as expected.



IUF EoS has dUrca threshold at $n_B = 4n_0$

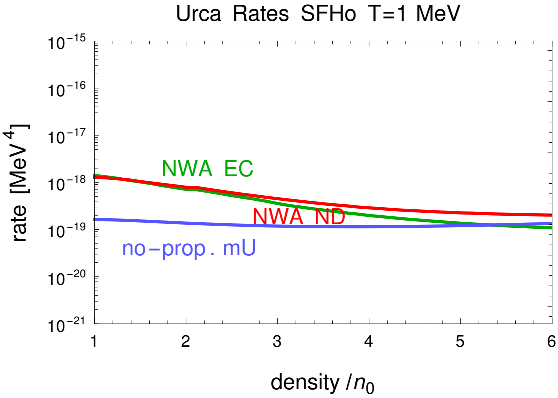
NWA for electron capture and neutron decay



Assumes "cold beta equilibrium" $\mu_n = \mu_p + \mu_e$ so the ND and EC rates should balance.

Below dUrca threshold, EC gets a bigger boost from thermal blurring of the Fermi surfaces than ND. NWA includes that.
Near the threshold there are corrections to the cold beta equil formula, arising from neutrino transparency.

EoS with no dUrca threshold

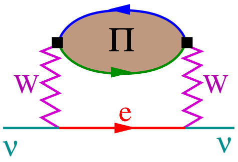


The whole rate comes from collisional broadening (mUrca). But traditional mUrca uses FS approx, cannot capture neutrino transparency effects that split ND from EC.

Generic strong interaction corrections

The neutron-proton loop is generalized to a *charged current hadronic correlator* $\Pi^{\mu\nu}(q) = \langle J^{\mu}_{\mathsf{weak}} J^{\dagger\,\nu}_{\mathsf{weak}} \rangle$ which includes all the strong interaction corrections.

$$\frac{dn_\nu}{dt} = \mathrm{Im} \left\{ \vphantom{\int} \right.$$



$$\left. \vphantom{\int} \right\}$$

Generic strong interaction corrections

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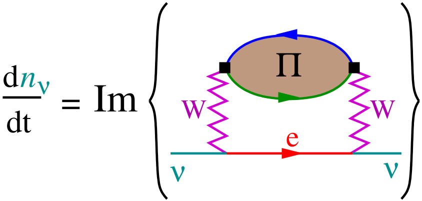

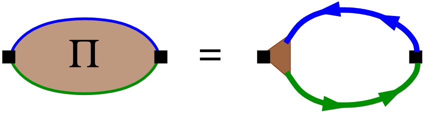

Skeleton expansion for Π in terms of full (1PI) vertex (brown triangle) and full nucleon propagators

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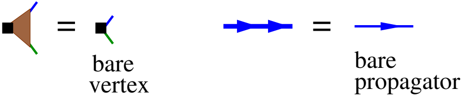
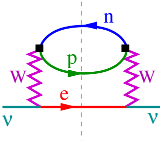
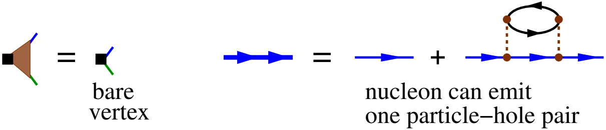
Approximations to the correlator

dUrca

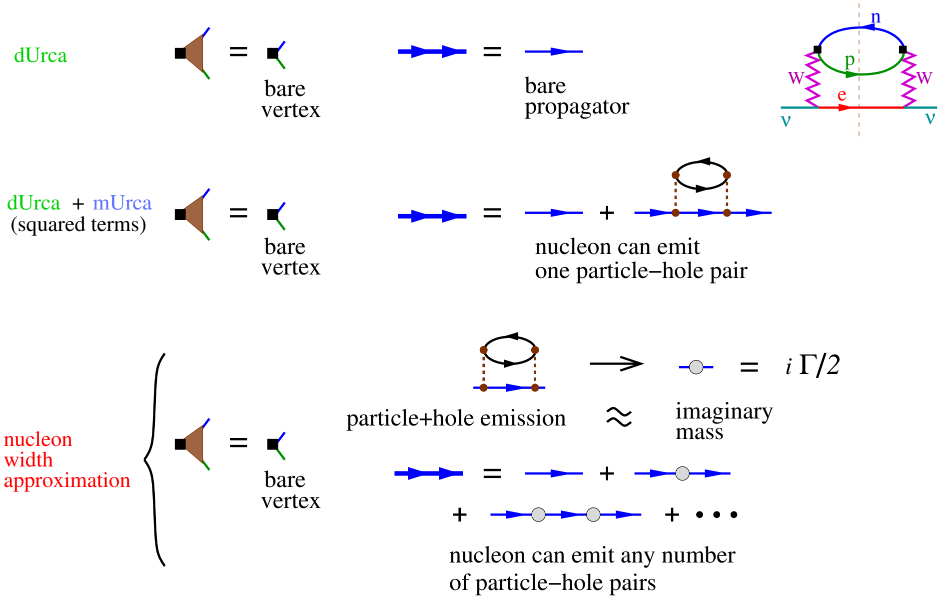




dUrca + mUrca
(squared terms)

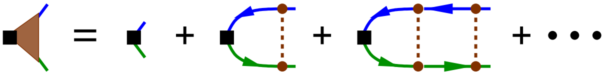


Approximations to the correlator



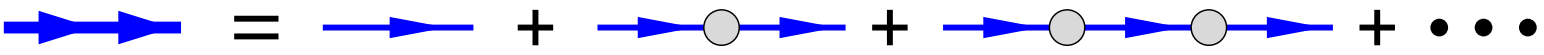
Beyond the nucleon width approx

(1) Vertex



RPA approx for vertex correction

(2) Nucleon Propagator



Schwinger–Dyson for full propagator, but with improved self–energy: