Dissipation in neutron star mergers

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muses
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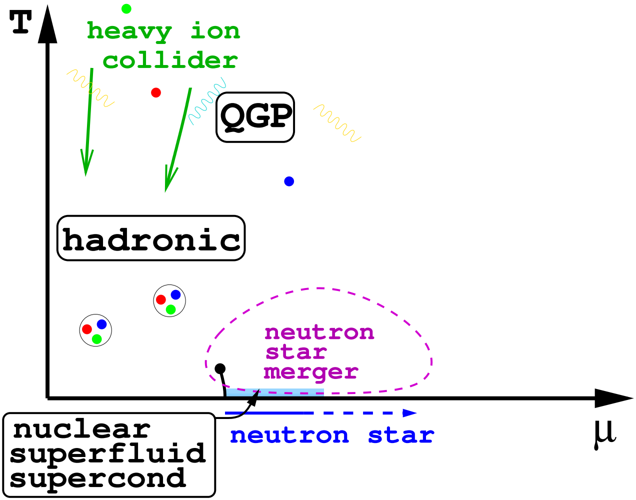
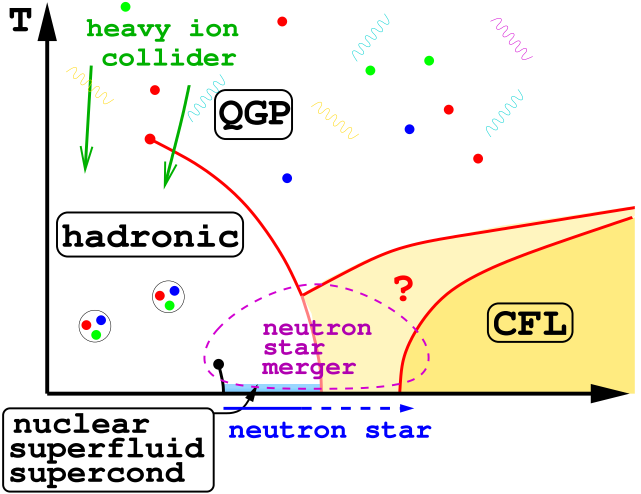
QCD Phase diagram
(observed)



Conjectured QCD Phase diagram



heavy ion collisions: deconfinement crossover and chiral critical point

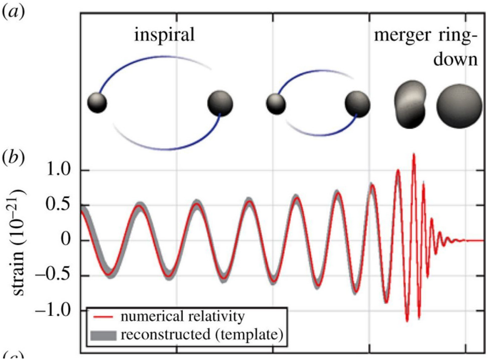
neutron stars: quark matter core?

neutron star mergers: dynamics of warm and dense matter

Observing mergers: prediction

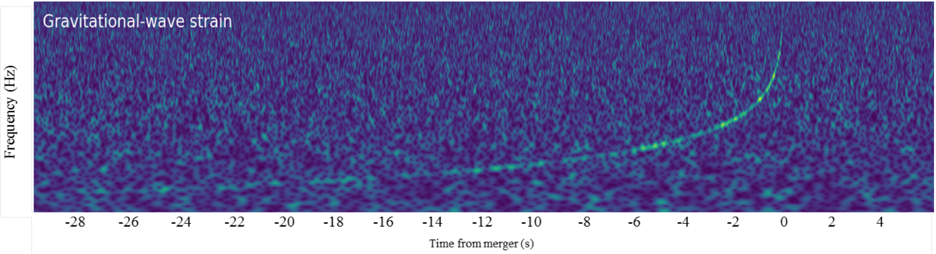
To use mergers as a probe of dense matter we need to perform simulations that incorporate the relevant microscopic physics.

E.g. to predict the gravitational wave signal



Observing mergers: data

LIGO Data from the event GW170817



Omega Scan of LIGO's First Neutron Star Merger Detection

We hope that future gravitational wave detectors such as Einstein Telescope or Cosmic Explorer will "hear" the inspiral for longer, and the merger and ringdown.

For now: work on making *accurate* predictions

Outline

- Neutron star mergers are like experiments that probe the properties of dense matter. People mostly talk about the *Equation of State*.
- Also potentially important: **Out-of-equilibrium phenomena**
  - Flavor equilibration — bulk viscosity
  - Thermal equilibration — thermal conductivity
  - Shear flow equilibration — shear viscosity

  etc

  Better than the equation of state for probing phase structure!

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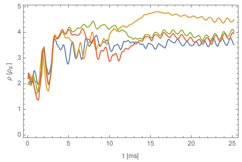
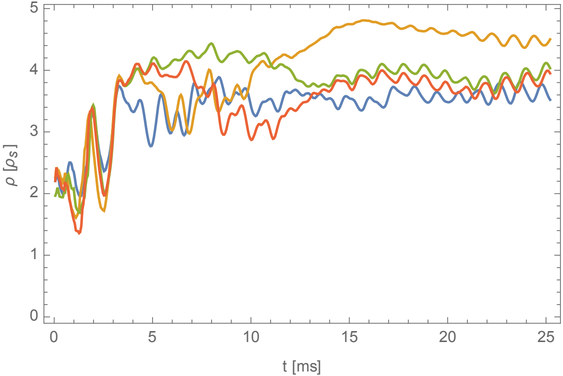
  Better than the equation of state for probing phase structure!
- Flavor equilibration: is it important in mergers?
  - relaxation time for the proton fraction
  - *Critical equilibration*: when relaxation should be included in the dynamics
  - physical manifestations: bulk viscosity and sound attenuation



Density oscillations in mergers

Density vs time for tracers in merger

Flavor equilibration neglected



Tracers (co-moving fluid elements) show dramatic density oscillations, especially in the first 5 ms.

Amplitude: up to 50%
Period: 1–2 ms
Freq: ~ 1 kHz

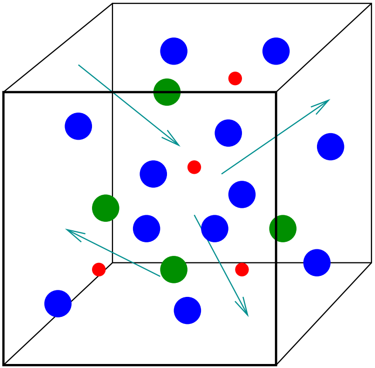
Do density oscillations drive the system out of flavor equilibrium?

Does flavor equilibration affect the oscillations?

The nuclear matter fluid

neutrons: dominant constituent
protons: small fraction
electrons: maintaining local neutrality
neutrinos: *not thermally equilibrated?*

Generic fluid element



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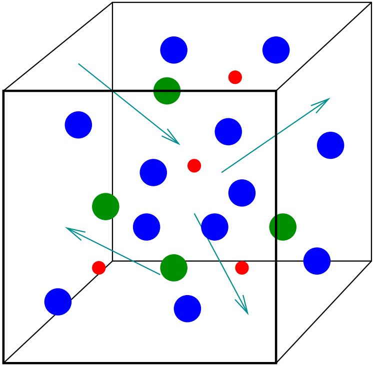

Fluid is described by 3-4 parameters:

$\boxed{n_B} = n_{\mathsf{n}} + n_{\mathsf{p}}$	baryon density
$\boxed{T}$	temperature
$\boxed{x_p} = n_{\mathsf{p}}/n_B$	proton fraction
($\boxed{x_L} = n_L/n_B$ [if neutrinos are trapped]	lepton fraction)

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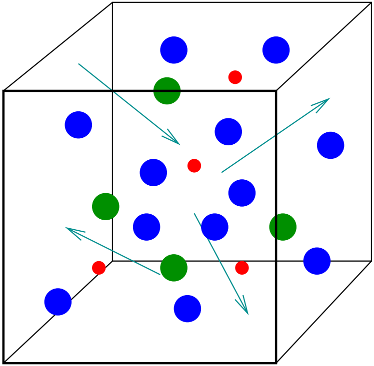

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[if neutrinos are trapped]

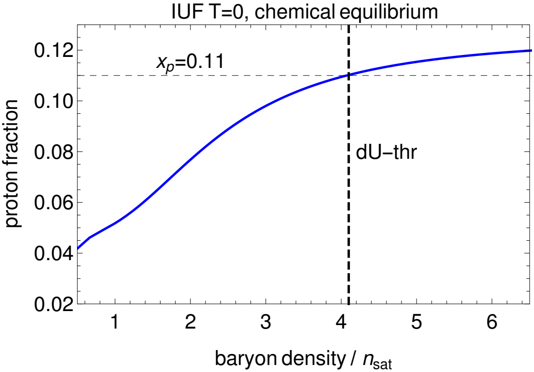
Equation of state relates these to relevant quantities: pressure, energy density etc,

$$p(n_B, T, x_p, x_L)$$
$$\varepsilon(n_B, T, x_p, x_L)$$
$$\ldots$$

Density oscillations and beta equilibration

Each fluid element relaxes to the *equilibrium proton fraction* $x_p^{\text{eq}}(n_B,T)$ via weak interactions.

$x_p^{\text{eq}}(n_B,T)$ is determined by properties of the strong interaction (nuclear symmetry energy) and the requirement of electrical neutrality.



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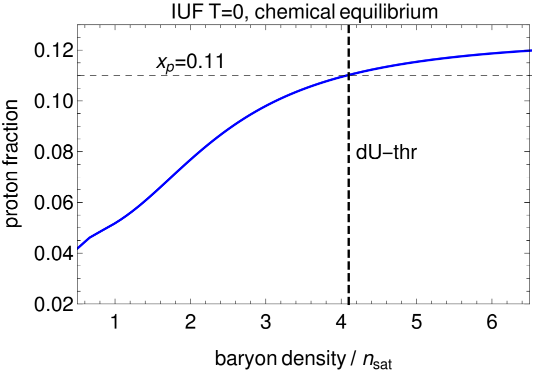

So when you compress nuclear matter, the proton fraction wants to change.

But this doesn't happen instantaneously!

Density oscillations can drive the system away from flavor ("beta", "chemical", "isospin") equilibrium.

Fast and slow equilibration

- Fluid element undergoes density oscillation of angular frequency ω

$$n_B(t) = \bar{n} + \delta n \cos(\omega t)$$

- Proton fraction relaxes to equilibrium at *relaxation rate* $\boldsymbol{\gamma}(n_B, T)$

$$\partial_t x_p = -\boldsymbol{\gamma}\big(x_p - x_p^{\text{eq}}(n_B, T)\big)$$

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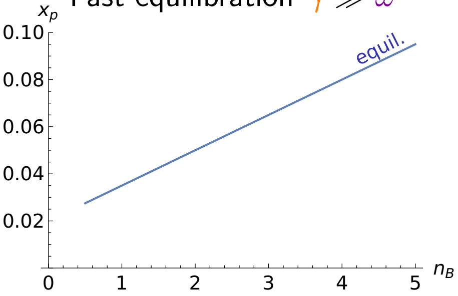

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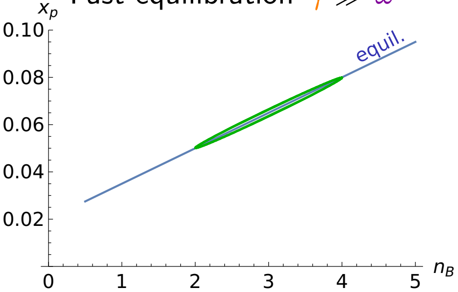

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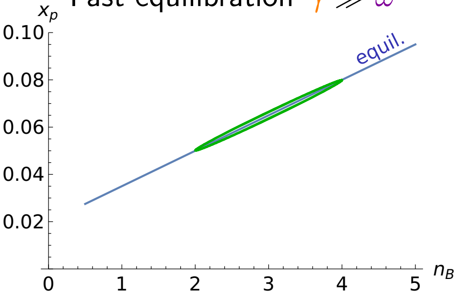

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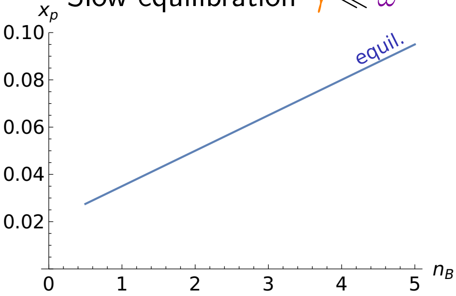

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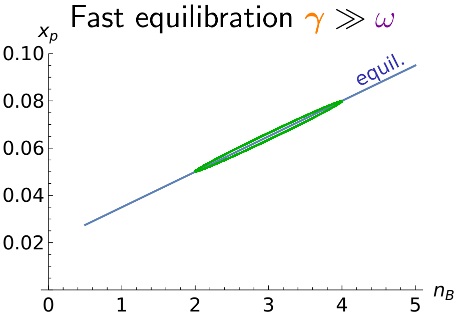

Fast equilibration	Slow equilibration
Proton fraction stays equilibrated	Proton fraction hardly changes
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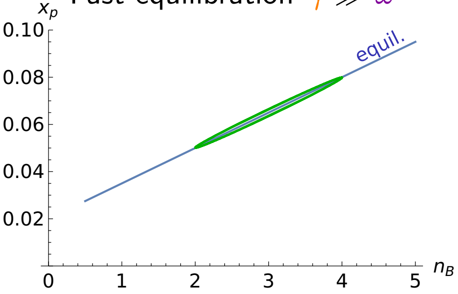

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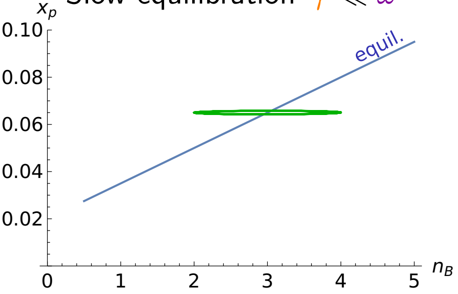

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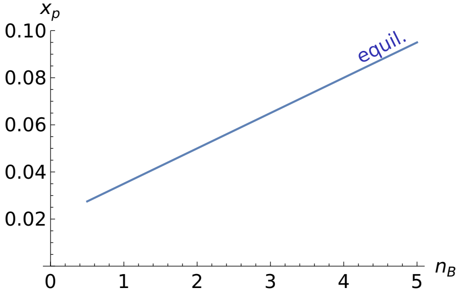
What happens if $\gamma \sim \omega$?

Critical equilibration

Critical equilibration $\gamma = \omega$



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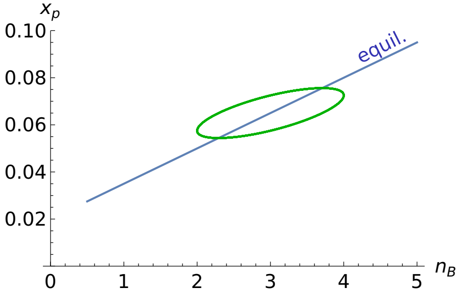

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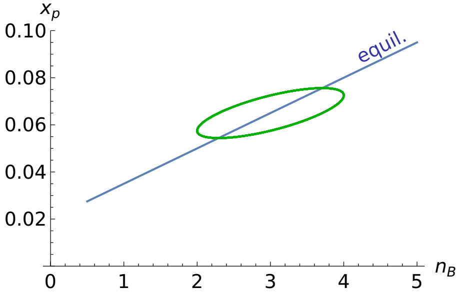

- The proton fraction $x_p(t)$ depends on *recent history*, not just $n_B(t)$.
- Should include the relaxation equation in the fluid dynamics

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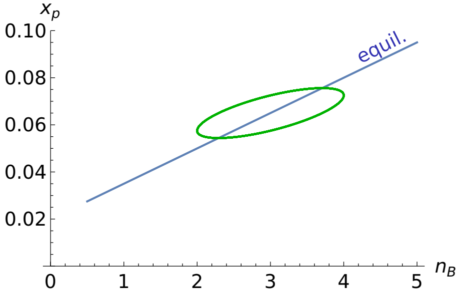

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Other features of critical equilibration:

- Maximal phase lag between density and proton fraction
- Maximal bulk viscosity $\Rightarrow$ Maximal damping of density oscillations

Is there critical equilibration in mergers?

Critical equilibration ($\gamma = \omega$) in mergers?

Frequency for typical density oscillations in a merger: $\boxed{\omega \approx 2\pi \times 1\,\text{kHz}}$

Relaxation rate $\gamma(n_B, T)$ for proton fraction: determined by weak interaction "Urca processes" in which neutrinos play an essential role.

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Relaxation rate $\gamma(n_B,T)$ for proton fraction: determined by weak interaction "Urca processes" in which neutrinos play an essential role.

We can calculate the relaxation rate in two limiting cases:

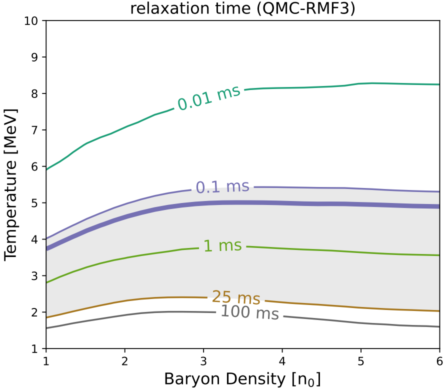
Urca process	neutrino-transparent	neutrino-trapped
neutron decay	$n \to p + e^- + \bar{\nu}_e$	$\nu_e + n \to p + e^-$
electron capture	$p + e^- \to n + \nu_e$	$p + e^- \to n + \nu_e$

When is $\gamma(n_B,T)$ comparable to the $2\pi\times 1\,\text{kHz}$ timescale?

At what density and temperature?

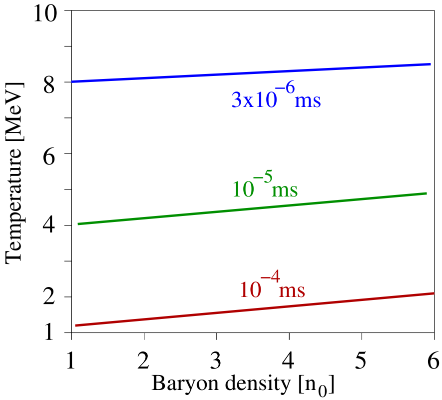
Proton fraction relaxation time $\tau = 1/\gamma$,

neutrino-transparent



Alford, Haber, Zhang `arXiv:2306.06180`

neutrino-trapped



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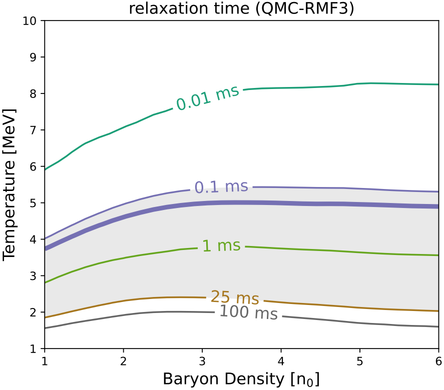

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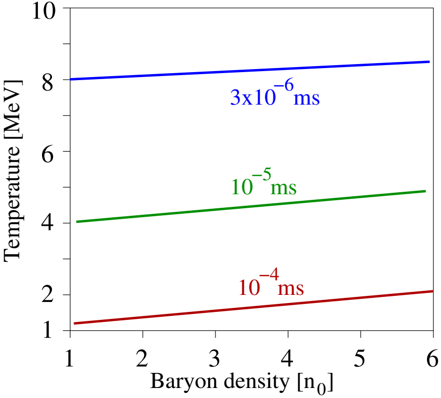

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- Relaxation is faster at higher temperatures, insensitive to density
- neutrino-trapped matter: relaxation is very fast
- neutrino-transparent matter: relaxation on merger timescales!
- Thick contour shows critical equilibration, where $\tau = 1\,\text{ms}/2\pi$

Conclusions so far

- Neutrino-trapped matter:
  proton fraction relaxes *quickly*, in microseconds at $T \geqslant 1\,\text{MeV}$. Only merger simulations with very short timesteps would need to include this process.
- Neutrino-transparent matter:
  at $T \sim 2$ to $5\,\text{MeV}$, proton fraction relaxes on the same timescale as the merger dynamics. Critical equilibration!
  Proton fraction equilibration is part of the dynamics.

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In reality, neutrinos in mergers have some non-thermal distribution with an energy-dependent mean free path. Need to develop tools to deal with this.

If critical equilibration (relaxation time $\approx$ oscillation period) occurs in mergers, are there physical consequences?

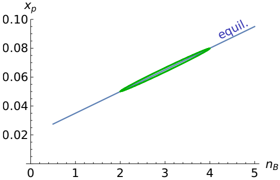
Bulk viscosity: phase lag in system response

Some property of the material (proton fraction) takes time to equilibrate

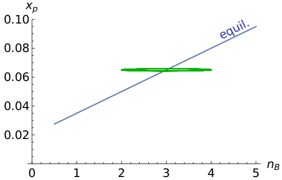
Baryon density n (fluid element volume V) goes out of phase with applied pressure P

$$\text{Dissipation} = -\int P\, dV$$

Fast equilibration

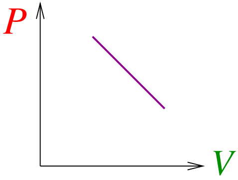


Slow equilibration



$\Rightarrow$

No dissipation



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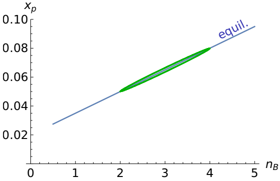

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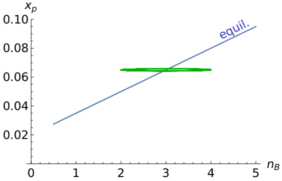

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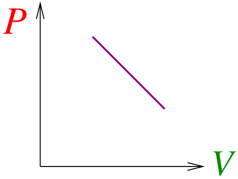

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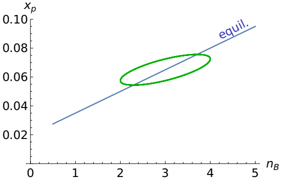
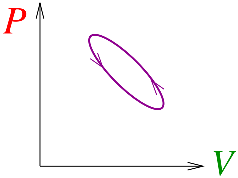


$\Rightarrow$

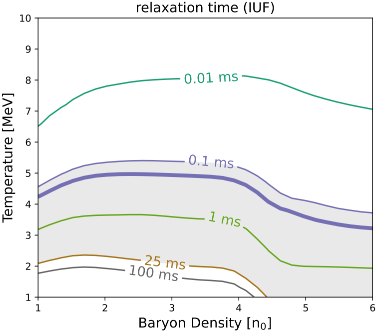
Max dissipation

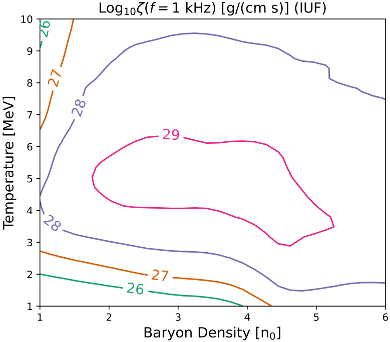


Resonant peak in bulk viscosity

(neutrino-transparent)

Critical equilibration ($\gamma = \omega$) $\Rightarrow$ Maximum bulk viscosity





- *Non-monotonic T-dependence:* bulk viscosity reaches a maximum at $T \sim 5\,\text{MeV}$ because that's where $\gamma(T) \approx 2\pi\times 1\,\text{kHz}$
- Not very sensitive to density, except dUrca threshold at $n_B = 4.1 n_{\text{sat}}$

Bulk viscosity: a resonant phenomenon

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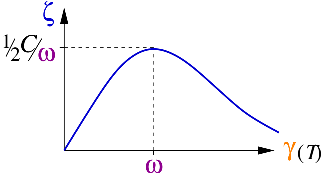

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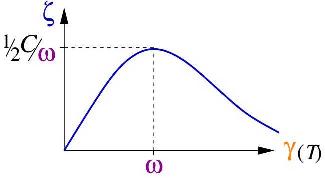

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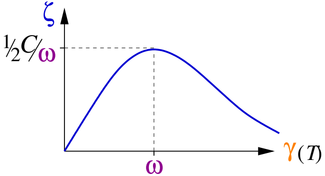

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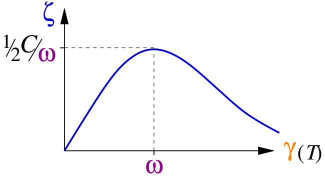

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  System does not try to equilibrate: Proton fraction fixed.
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- Critical equilibration: $\omega = \gamma \;\Rightarrow$ maximum phase lag between pressure and density $\Rightarrow$ maximum dissipation

Density oscillation damping time τ_{damp}

Different from proton fraction relaxation time τ

Density oscillation of amplitude Δn at angular freq ω:

$$n(t) = \bar{n} + \Delta n \cos(\omega t)\ \mathrm{e}^{-t/2\tau_{\text{damp}}}$$

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Energy of density oscillation:
(K = nuclear incompressibility)

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Density oscillation damping time τ_{damp}

Different from proton fraction relaxation time τ

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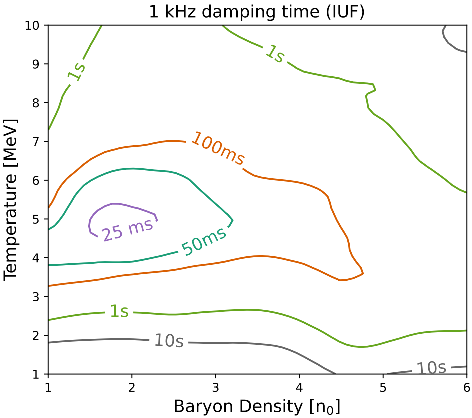
Damping Time: $\tau_{\mathsf{damp}} = \dfrac{E_{\mathrm{comp}}}{W_{\mathrm{comp}}} = \dfrac{K\bar{n}}{9\omega^2\,\zeta}$

Damping (sound attenuation) due to flavor equilibration
is important in mergers if $\tau_{\mathsf{damp}} \lesssim 20\,\mathrm{ms}$

Damping time (neutrino-transparent)

$$\tau_{\text{damp}} = \frac{K\bar{n}}{9\omega^2\,\zeta}$$

- *Non-monotonic T-dependence:* damping is fastest at $T \sim 5\,$MeV because bulk viscosity peaks there.



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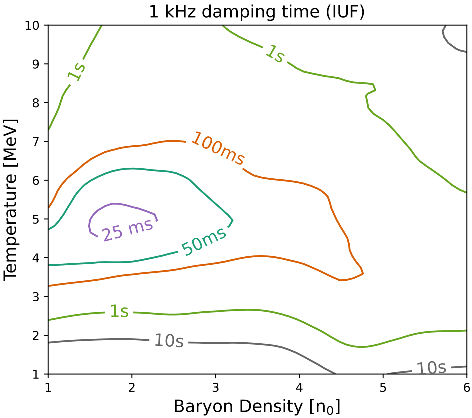

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  Baryon density $\bar{n}$ and incompressibility K are both increasing.
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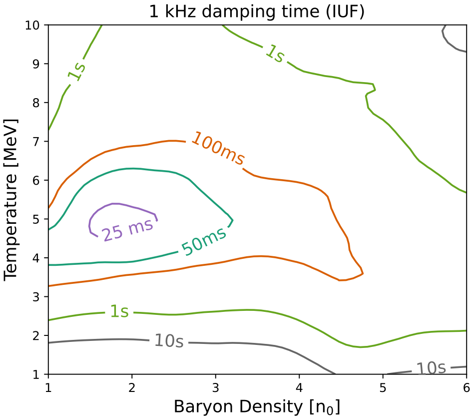

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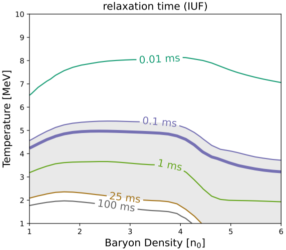
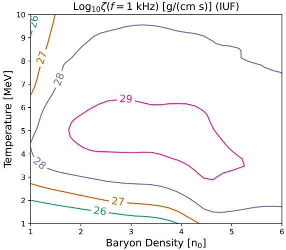
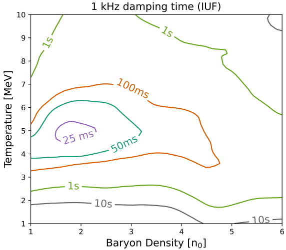
  Oscillations carry more energy $\Rightarrow$ slower to damp
- Damping of a $1\,\text{kHz}$ density oscillation can occur on merger timescales

Recap: flavor equilibration in nuclear matter
(neutrino-transparent)

Critical equilibration ($\gamma = \omega$) $\Rightarrow$ Maximum bulk viscosity $\Rightarrow$ Fastest damping

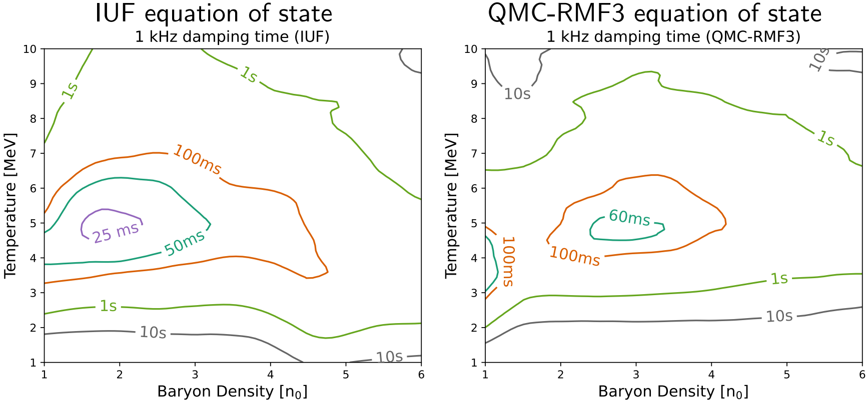






Bulk viscosity and damping of oscillations are strongest at $T \sim 5\,\text{MeV}$ because that's where $\gamma(T) \approx 2\pi \times 1\,\text{kHz}$

Two different EoSes



The damping time for density oscillations is shortest around $T \sim 5,$MeV, independent of the EoS.

In neutrino-transparent matter, damping time is short enough to be relevant for mergers, especially at low density.

Bulk viscosity in neutrino-trapped regime

$$\zeta = C\frac{\gamma}{\gamma^2+\omega^2}$$

Relaxation is much faster

Susceptibility C is smaller

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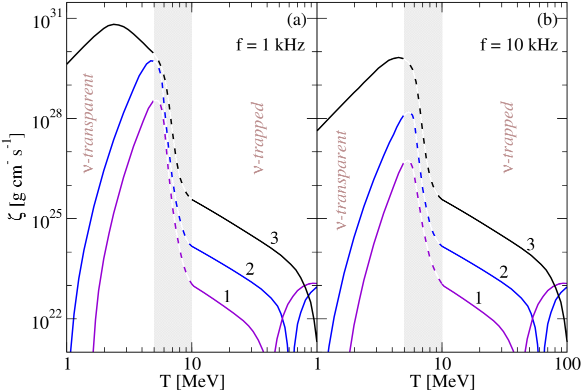


Plot shows bulk viscosity,

$T < 5\,\text{MeV}$:
neutrino-transparent
$n \to p\, e^-\, \bar{\nu}_e$
$p\, e^- \to n\, \nu_e$

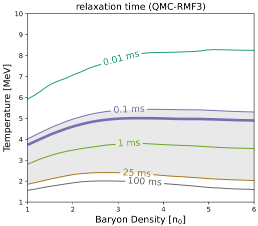
$T > 10\,\text{MeV}$:
neutrino-trapped
$\nu_e\, n \leftrightarrow p\, e^-$

Bulk viscosity is *lower* in hot matter ($T \gtrsim 5\,\text{MeV}$)
$\Rightarrow$ damping time is much longer.

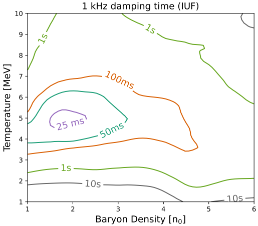
Summary

- Neutron star mergers probe the dynamical response of high-density matter on the millisecond timescale.
- In neutrino-transparent nuclear matter at $T \sim 2$ to $5\,\mathrm{MeV}$: *critical equilibration*.

  Proton fraction relaxes in milliseconds.



- Resultant bulk viscosity damps density oscillations in 20 to 100 ms



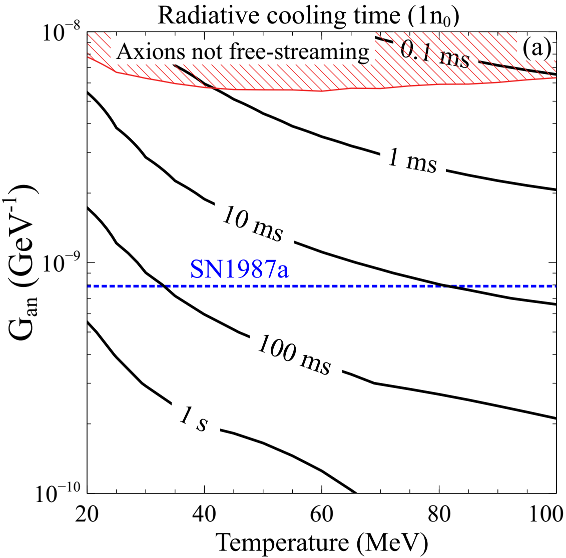
Important to include weak interactions in simulations

Next steps

- Beyond neutrino transparent/trapped:
  Flavor equilibration rates for arbitrary neutrino distributions
- Beyond npe:
  Flavor equilibration rates for other forms of matter.
  - Hyperonic: fast relaxation
  - Pion condensed, nuclear pasta, quark matter, etc
- Beyond bulk viscous damping:
  Other manifestations of flavor equilibration:
  - Heating
  - neutrino emission
- Beyond flavor equilibration:
  Thermal conductivity and shear viscosity may become significant in the neutrino-trapped regime if there are gradients of scale $\lesssim 100\,\mathrm{m}$.
- Beyond Standard Model physics?

Cooling by axion emission

Time for a hot region to cool to half its original temperature:



Harris, Fortin, Sinha, Alford
`arXiv:2003.09768`