Measurement-induced spectral/topological transitions

K. Mochizuki and R. Hamazaki,
Physical Review Letters **134**, 010410 (2025).

H. Oshima, K. Mochizuki, R. Hamazaki, and Y. Fuji,
Physical Review Letters **134**, 240401 (2025).

科研費 KAKENHI

ERATO 沙川情報エネルギー変換プロジェクト

Collaborators

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Take-home message

isolated quantum systems
governed by Hamiltonians

$$\hat{H}\,|\Psi_i\rangle = \varepsilon_i\,|\Psi_i\rangle$$

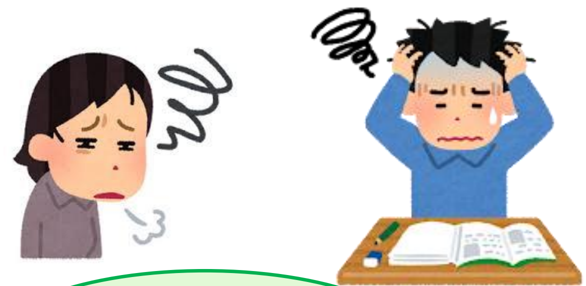
$|\psi\rangle$

spectrum,
ground state,
topology,
...

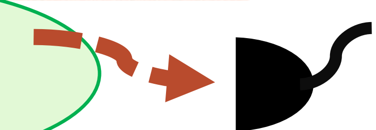
our work

monitored quantum systems
not governed by Hamiltonians

intriguing entanglement transitions
owing to measurements (discussed later)



$|\psi\rangle$



same concepts!

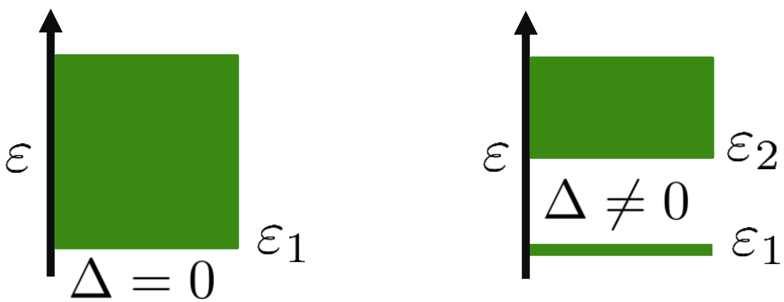
Outline

- Quantum phase transitions in isolated systems
- Measurement-induced transitions in monitored quantum systems
- Motivation and rough sketch of results
- Result-1: measurement-induced spectral transition in interacting spin systems
- Result-2: measurement-induced topological transition in non-interacting fermionic systems
- Summary

Ground-state phase transition in a spin chain

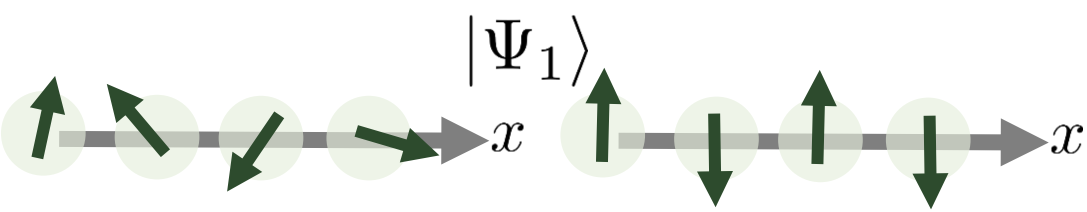
the spectral gap $\Delta = \varepsilon_2 - \varepsilon_1, \quad \varepsilon_i \le \varepsilon_{i+1}$

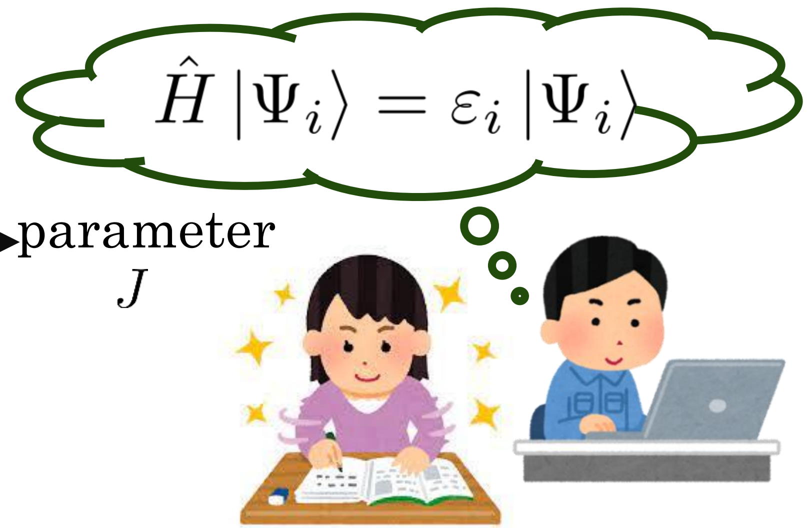
transition of the gap $\Longleftrightarrow$ ground-state phase transition



e.g. XXZ model

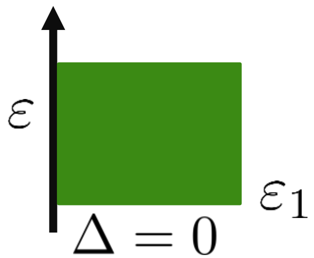
$$\hat{H} = \sum_x \hat{X}_x \hat{X}_{x+1} + \hat{Y}_x \hat{Y}_{x+1} + J \hat{Z}_x \hat{Z}_{x+1}$$

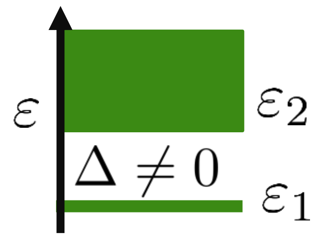




Entanglement entropy and phase transitions

the spectral gap $\Delta = \varepsilon_2 - \varepsilon_1, \quad \varepsilon_i \leq \varepsilon_{i+1}$
transition of the gap ⟷ ground-state phase transition





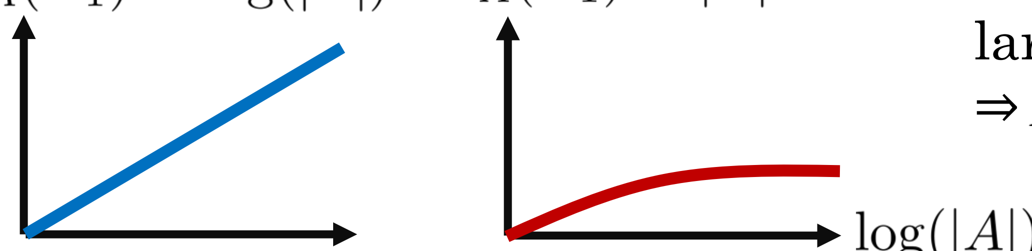
$$\hat{H} \left|\Psi_i\right\rangle = \varepsilon_i \left|\Psi_i\right\rangle$$

1D, e.g. XXZ model

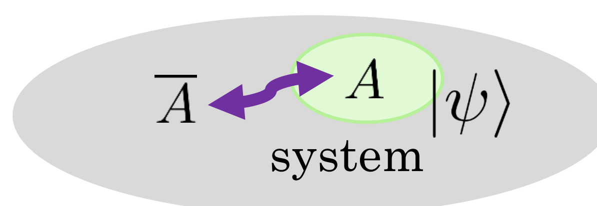
gapless phase logarithmic-law	gapped phase area-law

parameter

$S_A(\Psi_1) \propto \log(|A|)$ $\quad$ $S_A(\Psi_1) \propto |A|^0$



large $S_A(\psi)$
$\Rightarrow A$ and $\overline{A}$ are highly entangled



$$S_A(\psi) = -\mathrm{tr}_A \left(\rho_A(\psi) \log\left[\rho_A(\psi)\right]\right)$$
an indicator of how complex the state is

$$\rho_A(\psi) = \mathrm{tr}_{\bar{A}} \left(\left|\psi\right\rangle \left\langle\psi\right|\right)$$

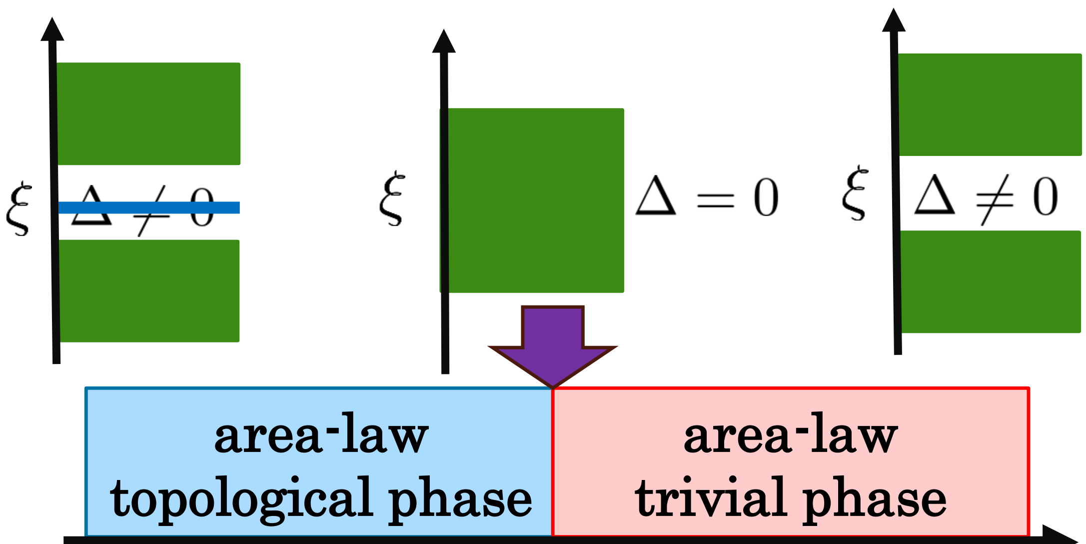
Topological transition between gapped phases

gap closing ⟷ topological phase transition

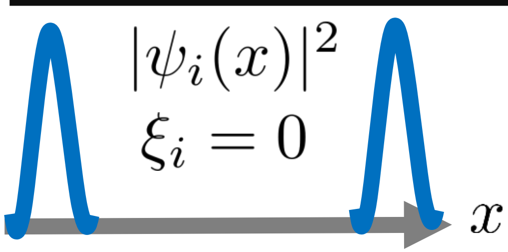
Kitaev chain: $\vec{c}^\dagger = (c_1^\dagger, \cdots, c_L^\dagger, c_1, \cdots, c_L)$

$$\hat{H} = \sum_x \hat{c}^\dagger_{x+1}\hat{c}_x + \hat{c}_{x+1}\hat{c}_x + h.c. + \mu \hat{c}^\dagger_x \hat{c}_x = \vec{c}^\dagger H \vec{c}$$

$$H\left|\psi_i\right\rangle = \xi_i \left|\psi_i\right\rangle$$



parameter μ



Majorana edge mode
in the topological phase

c.f. $\Delta \neq 0 \Rightarrow$ area law

M. B. Hasting and T. Koma,
Commun. Math. Phys. **265**, 781 (2006).

Bulk-edge correspondence

Majorana edge mode under the open boundary condition (OBC)

⇕

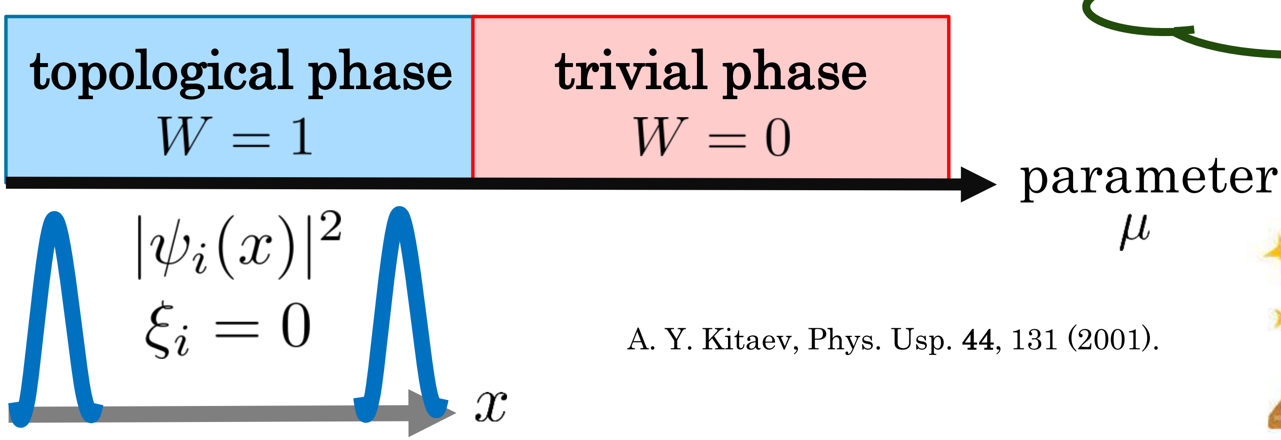
non-zero topological number W under PBC/APBC

$$\hat{H} = \sum_x \hat{c}^\dagger_{x+1}\hat{c}_x + \hat{c}_{x+1}\hat{c}_x + h.c. + \mu\hat{c}^\dagger_x\hat{c}_x = i\vec{\gamma}^\dagger H\vec{\gamma}$$

Z_2 topological number computed from the Hamiltonian:

$$(-1)^W = \mathrm{sign}\left(\mathrm{Pf}\left[H_{\mathrm{PBC}}\right]\mathrm{Pf}\left[H_{\mathrm{APBC}}\right]\right)$$

$$H\left|\psi_i\right\rangle = \xi_i\left|\psi_i\right\rangle$$



A. Y. Kitaev, Phys. Usp. **44**, 131 (2001).

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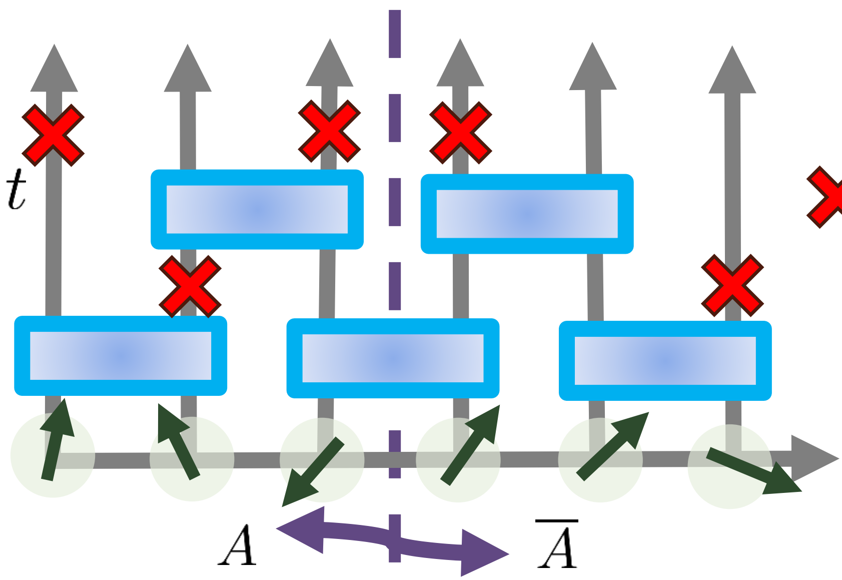
Monitored many-body spin systems



quantum gate $|\psi_t\rangle \to \hat{\mathsf{U}}_{x,x+1}|\psi_t\rangle$

competition ⇒ transition!

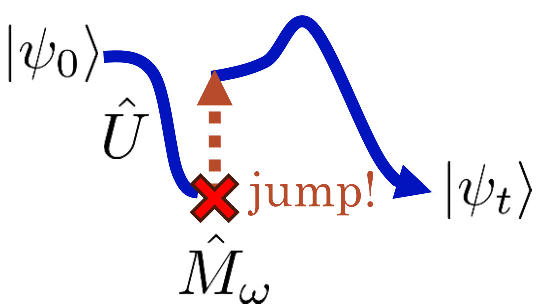
measurement on $\hat{Z}_x$ with probability p $\quad |\psi_t\rangle \to \dfrac{\hat{\mathsf{M}}_{\omega_x}|\psi_t\rangle}{\sqrt{q_t(\omega_x)}}$

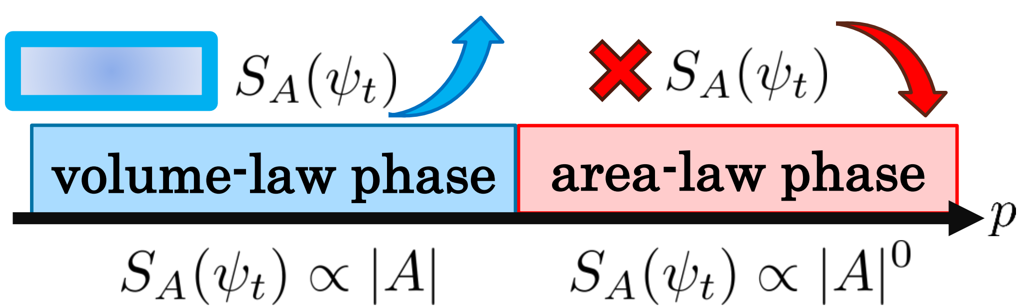
measurement operator: $\hat{\mathsf{M}}_{\omega_x=\pm} = \dfrac{\hat{I} \pm \hat{Z}_x}{2}$

$\omega_x = \pm$: random measurement outcome

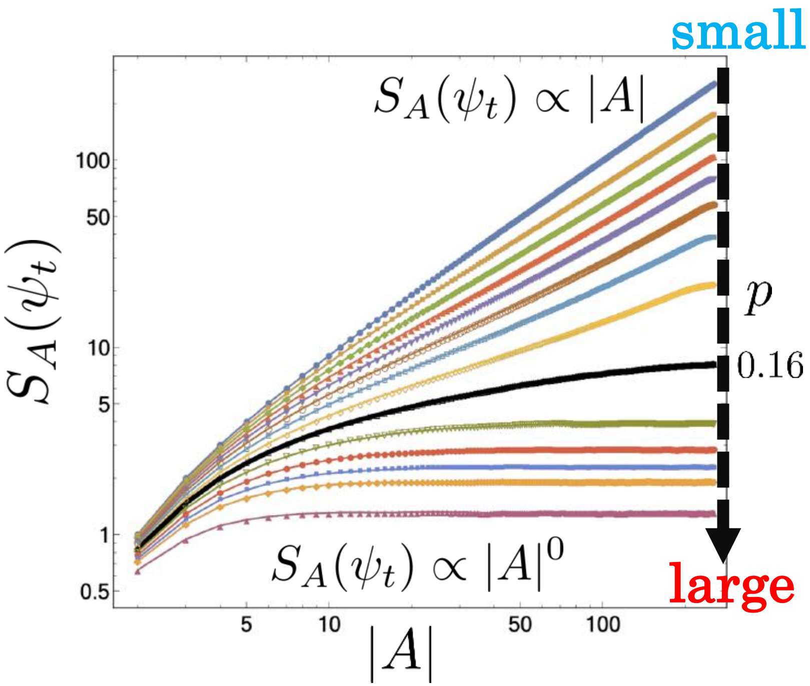
1/2 spin

Born probability: $q_t(\omega_x) = \langle\psi_t|\, \hat{\mathsf{M}}^\dagger_{\omega_x}\hat{\mathsf{M}}_{\omega_x}|\psi_t\rangle$



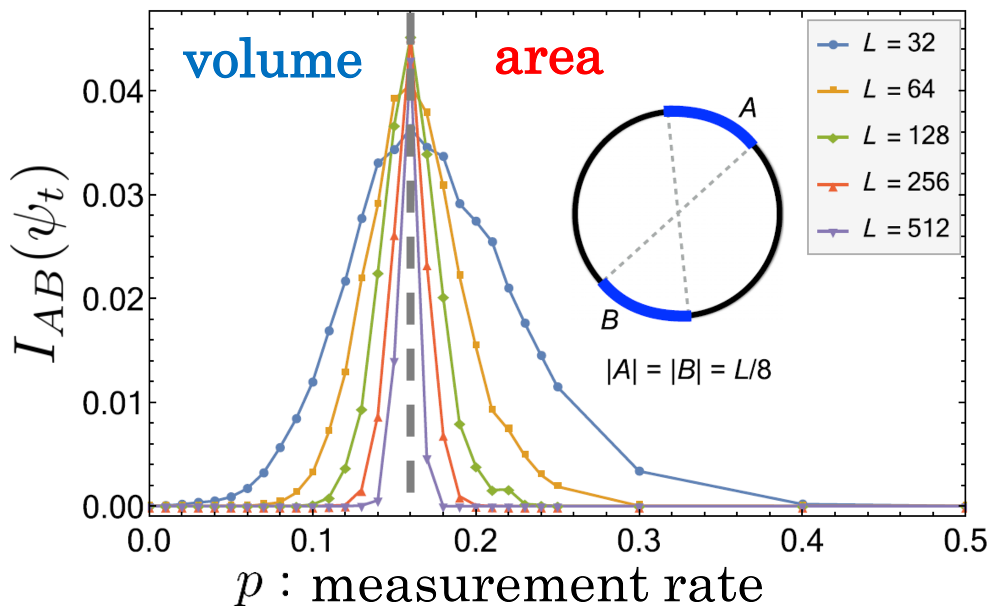


Measurement-induced entanglement transition



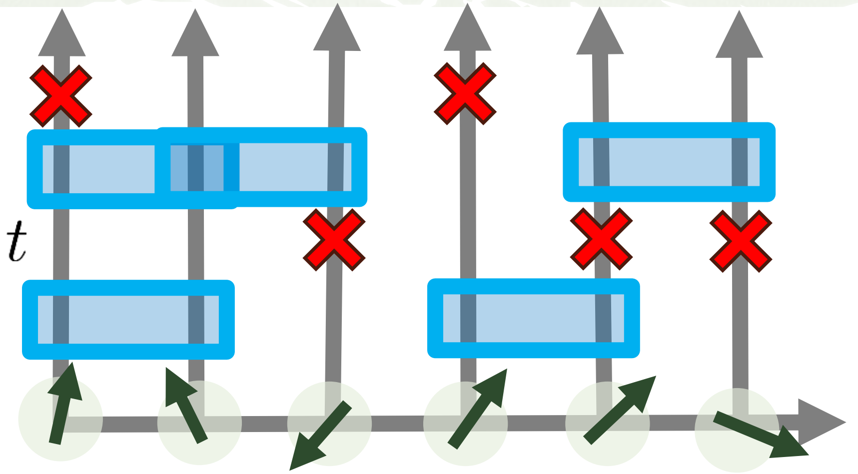
Y. Li et.al., PRB **100**, 134306 (2019).

$$I_{AB}(\psi) = S_A(\psi) + S_B(\psi) - S_{AB}(\psi)$$



volume-area entanglement transition ⟺ peak of $I_{AB}(\psi_t)$

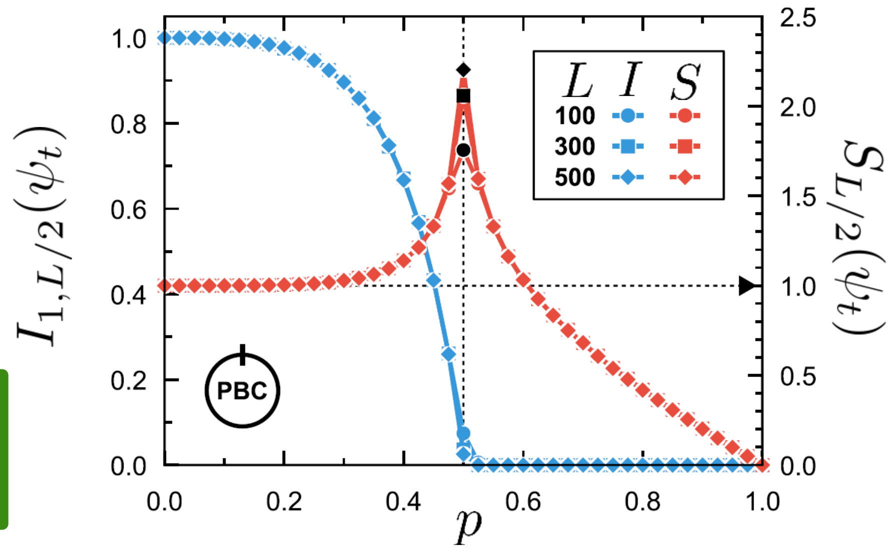
Transition in measurement-only circuit



measurement on $\hat{X}_x$ with probability p

measurement on $\hat{Z}_x \hat{Z}_{x+1}$ with probability $1 - p$

N. Lang and H. P. Buchler, PRB **102**, 094204 (2020).



Jordan-Wigner transformation
⇒ fermions with parity conservation

transition between two distinct area-law phases

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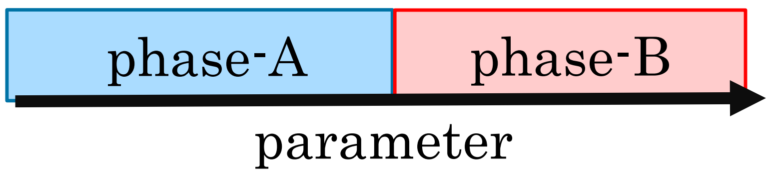
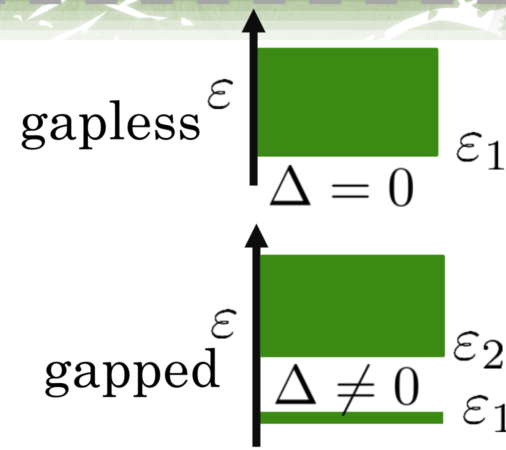
Comparison of isolated and monitored systems

isolated systems:

$\hat{H}\,|\Psi_i\rangle = \varepsilon_i\,|\Psi_i\rangle$

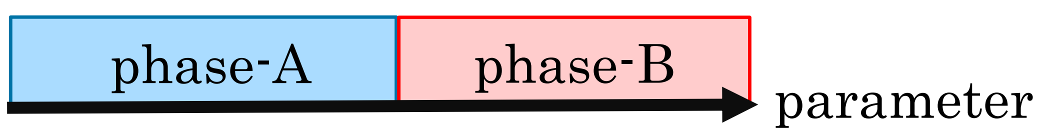
- entanglement scaling
- spectrum
- topological invariant
- …





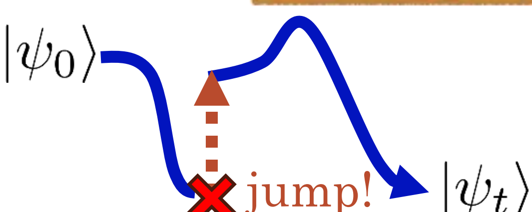
monitored systems:

- entanglement scaling
- ? spectrum
- ? topological invariant



entanglement transition ↔ spectrum/topology ??

Conventional spectral analysis seems difficult



temporal randomness due to measurement

no static generator $|\psi_t\rangle \neq e^{-i\hat{H}t}\,|\psi_0\rangle$

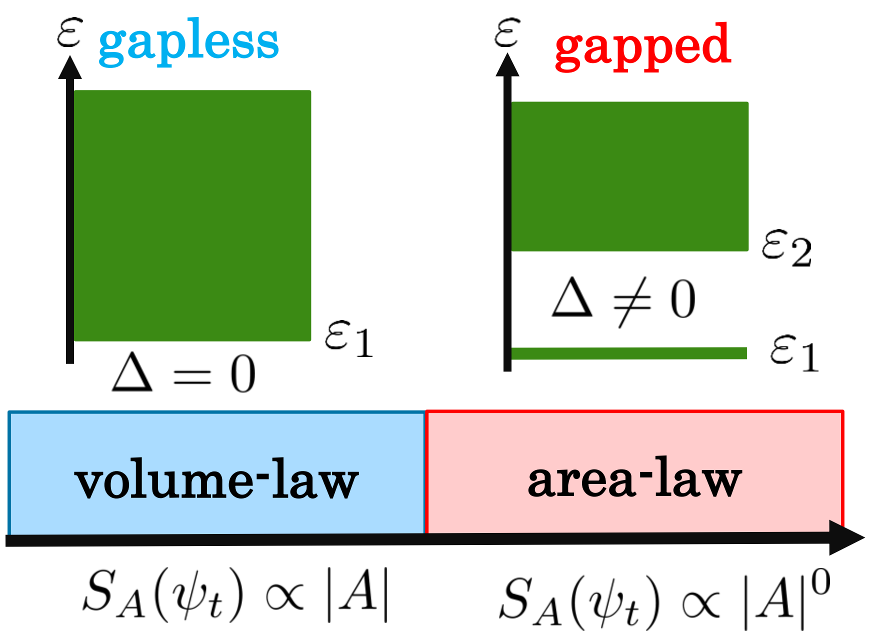
Objective, results

Phys. Rev. Lett. **134**, 010410 (2025).
Phys. Rev. Lett. **134**, 240401 (2025).

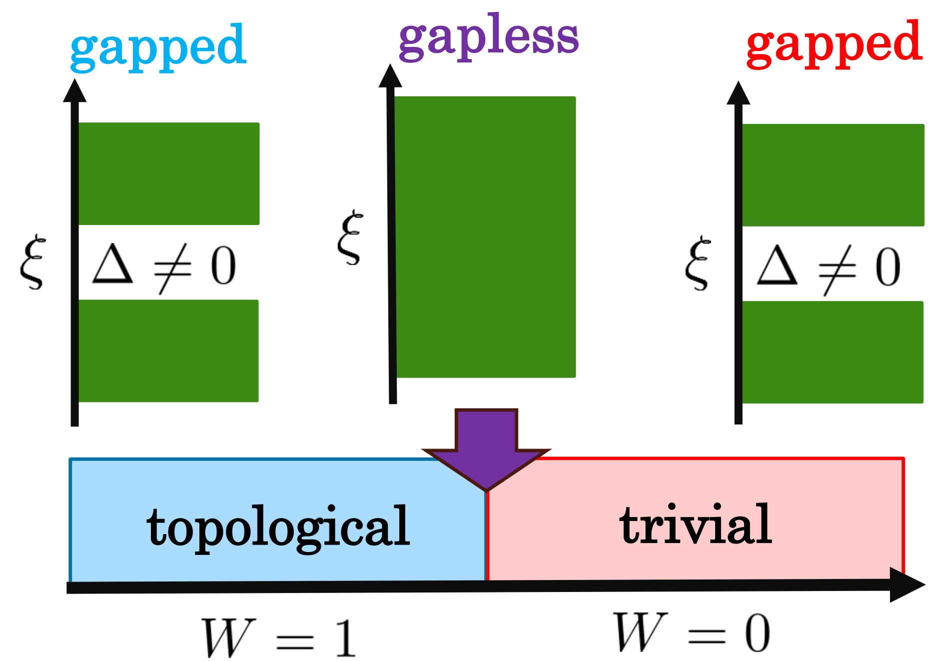
Measurement-induced entanglement transitions are related to some spectral/topological features??

analysis of the Lyapunov spectrum → **YES**

1. interacting spin system



2. non-interacting fermionic system



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Model: interacting hybrid circuit

Ken Mochizuki
Ryusuke Hamazaki
PRL **134**, 010410 (2025).

random nonunitary dynamics:
many trajectories conditioned on measurement outcomes

$\boldsymbol{\omega}_t = (\omega_1, \cdots, \omega_{t-1}, \omega_t)$

$$\boxed{|\psi_{\boldsymbol{\omega}_t}(\eta)\rangle \propto \hat{M}_{\omega_t}(\eta)\hat{U}_t \cdots \hat{M}_{\omega_1}(\eta)\hat{U}_1 |\psi_0\rangle}$$

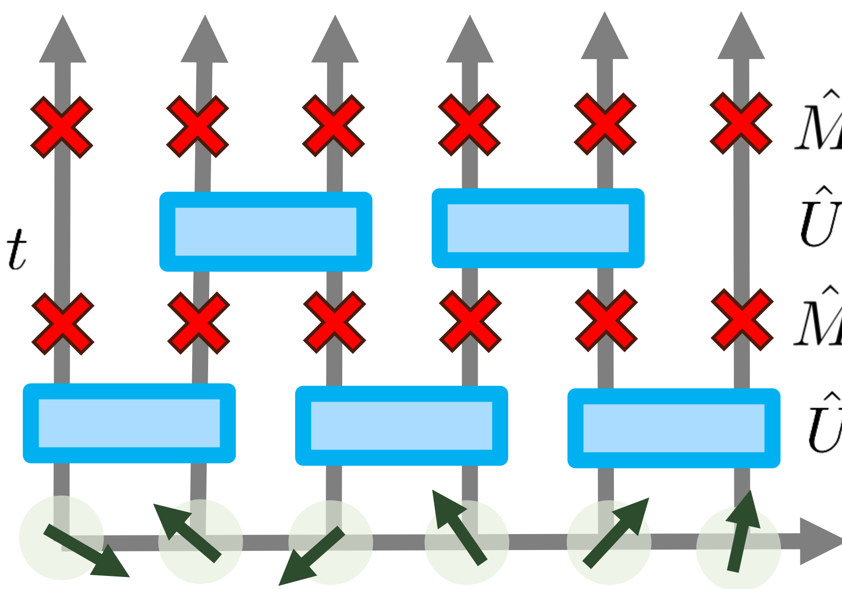
Born probability:

$$P_\eta(\omega_t|\boldsymbol{\omega}_{t-1}) = \left|\hat{M}_{\omega_t}(\eta)\hat{U}_t |\psi_{\boldsymbol{\omega}_{t-1}}(\eta)\rangle\right|^2$$

generalized measurement

$$\hat{\mathsf{M}}_{\omega_{t,x}=\pm}(\eta) \propto \hat{I} \pm \eta \hat{Z}_x$$

η : strength of measurement



$\hat{M}_{\omega_{t=2}}(\eta)$

$\hat{U}_{t=2}$

$\hat{M}_{\omega_{t=1}}(\eta)$

$\hat{U}_{t=1}$

$x = 1, 2, \cdots, L$

$\omega_{t,x} = \pm$: measurement outcomes

$$\hat{M}_{\omega_t}(\eta) = \prod_{x=1}^{L} \hat{\mathsf{M}}_{\omega_{t,x}}(\eta)$$

$$\omega_t = (\omega_{t,x=1}, \cdots, \omega_{t,x=L})$$

quantum gate

1/2 spin

Lyapunov analysis

Ken Mochizuki
Ryusuke Hamazaki
PRL **134**, 010410 (2025).

$$|\psi_{\boldsymbol{\omega}_t}(\eta)\rangle \propto \hat{V}_{\boldsymbol{\omega}_t}(\eta)\,|\psi_0\rangle\,, \quad \hat{V}_{\boldsymbol{\omega}_t}(\eta) = \hat{M}_{\omega_t}(\eta)\hat{U}_t \cdots \hat{M}_{\omega_1}(\eta)\hat{U}_1$$

effective "Hamiltonian": $\hat{H}_{\boldsymbol{\omega}_t}(\eta) = -\frac{1}{2t}\log\left[\hat{V}_{\boldsymbol{\omega}_t}(\eta)\hat{V}^\dagger_{\boldsymbol{\omega}_t}(\eta)\right]$

intuitive picture

imaginary-time evolution: $|\psi_{\boldsymbol{\omega}_t}(\eta)\rangle \sim \exp[-\hat{H}_{\boldsymbol{\omega}_t}(\eta)t]\,|\psi_0\rangle$

$$\hat{H}_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle = \varepsilon^i_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle\,, \quad \varepsilon^i_{\boldsymbol{\omega}_t}(\eta) \le \varepsilon^{i+1}_{\boldsymbol{\omega}_t}(\eta)$$

spectral gap : $\Delta(\eta) = \varepsilon_2(\eta) - \varepsilon_1(\eta), \quad \varepsilon_i(\eta) = \lim_{t\to\infty}\varepsilon^i_{\boldsymbol{\omega}_t}(\eta)$

ground state : $|\Psi^1_{\boldsymbol{\omega}_t}(\eta)\rangle$

analogy to ground-state transitions in isolated quantum systems

$$\hat{V}^\dagger_{\boldsymbol{\omega}_t}(\eta)\hat{V}_{\boldsymbol{\omega}_t}(\eta)\,|\Phi^i_{\boldsymbol{\omega}_t}(\eta)\rangle = \exp\left[-2\varepsilon^i_{\boldsymbol{\omega}_t}(\eta)t\right]|\Phi^i_{\boldsymbol{\omega}_t}(\eta)\rangle$$

$$\hat{V}_{\boldsymbol{\omega}_t}(\eta) \simeq \sum_i \exp\left[-\varepsilon_i(\eta)t\right]|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle\langle\Phi^i_{\boldsymbol{\omega}_t}(\eta)| \simeq |\Psi^1_{\boldsymbol{\omega}_t}(\eta)\rangle\langle\Phi^1_{\boldsymbol{\omega}_t}(\eta)|$$

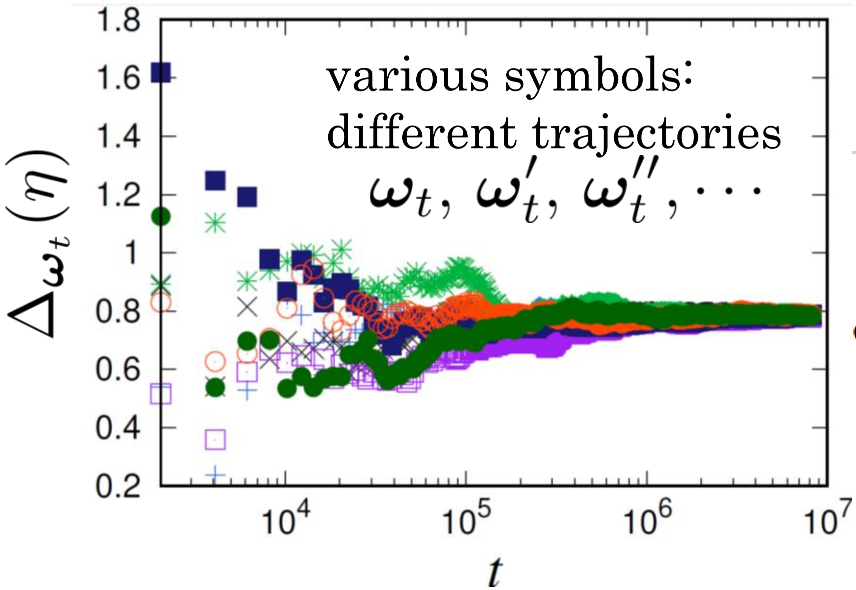
absorption to the ground state: $t \gg 1/\Delta(\eta) \to |\psi_{\boldsymbol{\omega}_t}(\eta)\rangle \simeq |\Psi^1_{\boldsymbol{\omega}_t}(\eta)\rangle$

Spectral gap

Ken Mochizuki
Ryusuke Hamazaki
PRL **134**, 010410 (2025).

$$\hat{H}_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle = \varepsilon^i_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle\,, \quad \varepsilon^i_{\boldsymbol{\omega}_t}(\eta) \le \varepsilon^{i+1}_{\boldsymbol{\omega}_t}(\eta)$$

spectral gap: $\Delta(\eta) = \lim_{t\to\infty} \Delta_{\boldsymbol{\omega}_t}(\eta),\ \Delta_{\boldsymbol{\omega}_t}(\eta) = \varepsilon^2_{\boldsymbol{\omega}_t}(\eta) - \varepsilon^1_{\boldsymbol{\omega}_t}(\eta)$

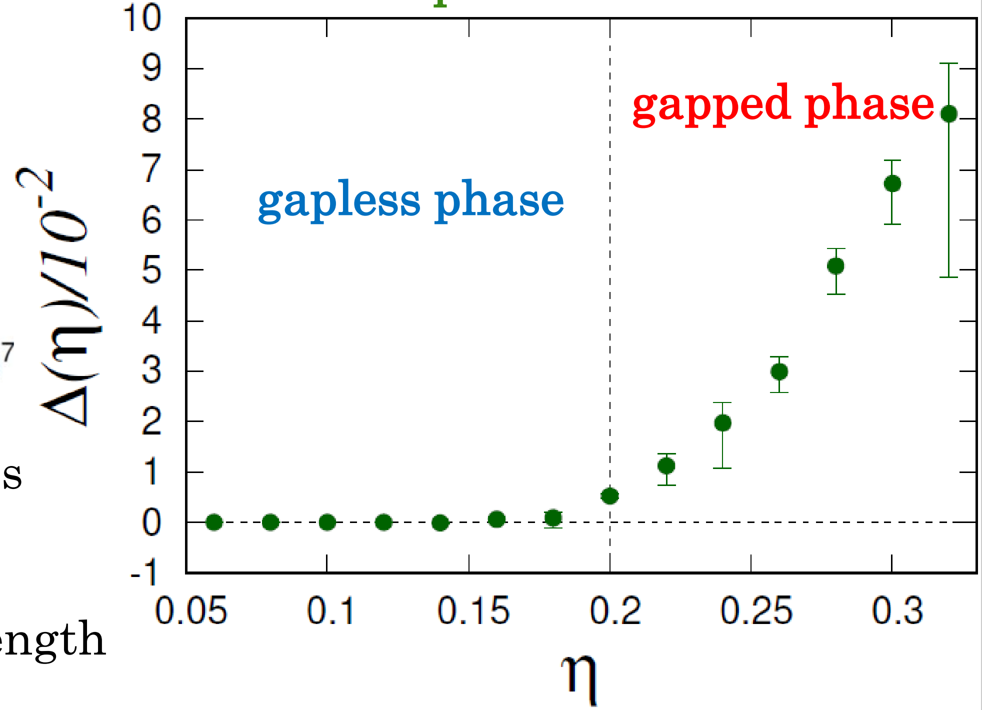


$\Delta(\eta)$: independent of trajectories

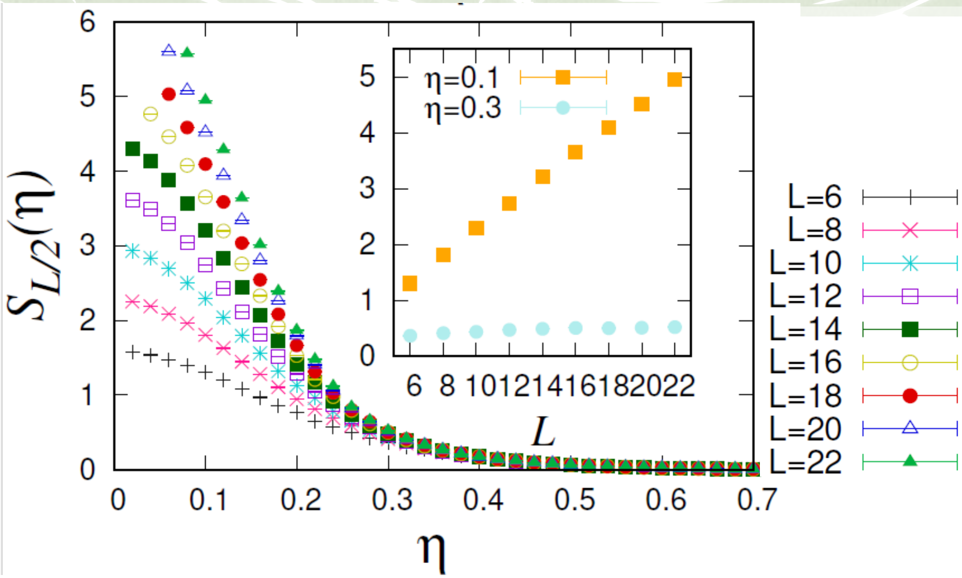
global quantity!

η : measurement strength

spectral transition!



Ground-state entanglement entropy

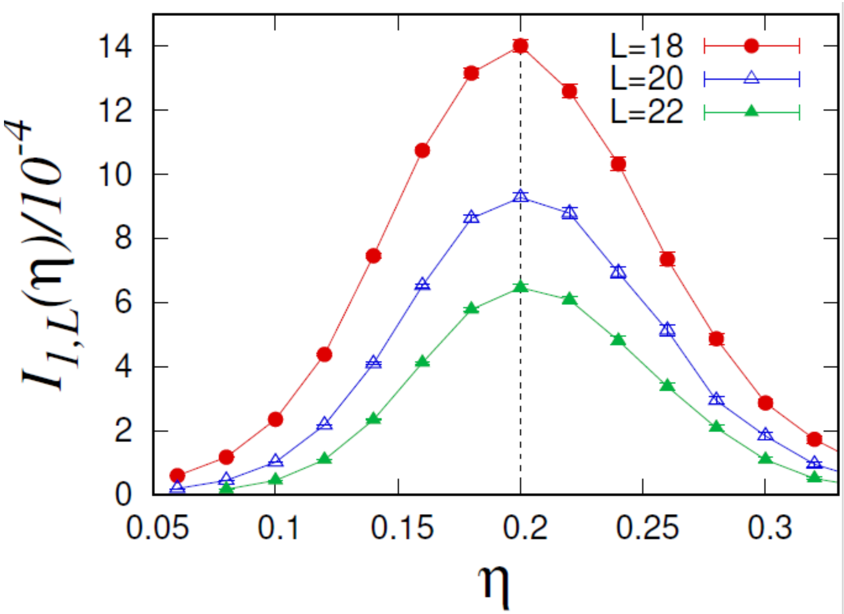


K. Mochizuki and R. Hamazaki
Phys. Rev. Lett. **134**, 010410 (2025).

$$I_{AB}(\psi) = S_A(\psi) + S_B(\psi) - S_{AB}(\psi)$$

peak of $I_{1,L}(\eta) = \mathbb{E}_t\left(I_{1,L}\left[\Psi^1_{\boldsymbol{\omega}_t}(\eta)\right]\right)$

→ phase transition!



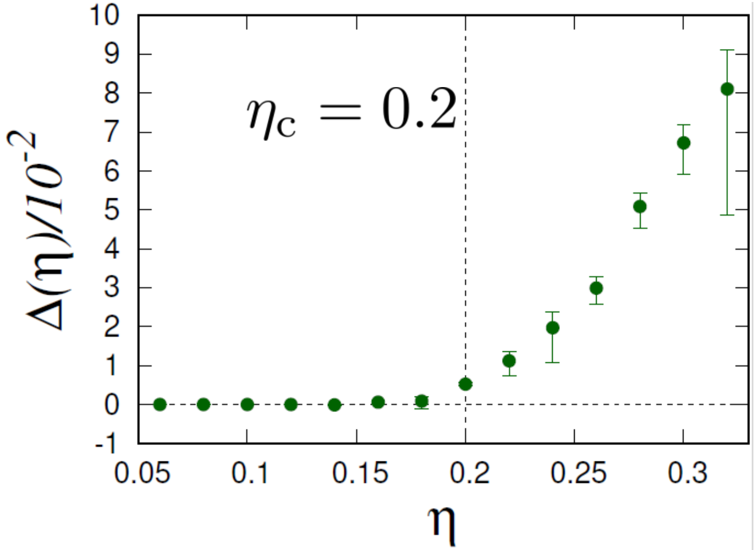
$$S_{L/2}(\eta) = \mathbb{E}_t\left(S_{L/2}\left[\Psi^1_{\boldsymbol{\omega}_t}(\eta)\right]\right)$$

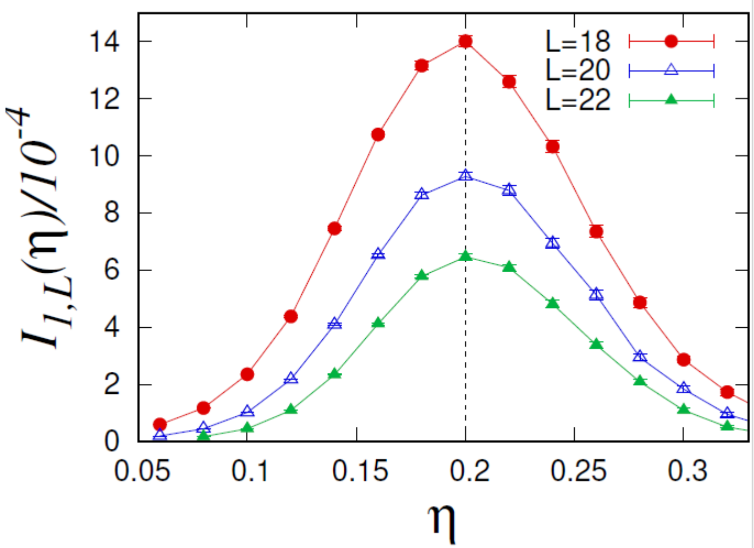
$$S_A(\psi) = -\mathrm{tr}_A\left(\rho_A(\psi)\log\left[\rho_A(\psi)\right]\right)$$

$\mathbb{E}_t$:time average $\quad \rho_A(\psi) = \mathrm{tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$

small η : volume law $S_{L/2}(\eta) \propto L$
large η : area law $S_{L/2}(\eta) \propto L^0$

Coincidence of the thresholds





$$\hat{H}_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle = \varepsilon^i_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle$$

gapless phase = volume-law phase

$\Delta(\eta) = 0 \qquad S_{L/2}(\eta) \propto L$

gapped phase = area-law phase

$\Delta(\eta) \neq 0 \qquad S_{L/2}(\eta) \propto L^0$

analogous to isolated quantum systems

concepts established in isolated systems
can be extended to monitored systems??

Ken Mochizuki, Ryusuke Hamazaki,
Physical Review Letters **134**, 010410 (2025).

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Model : measurement-only Majorana circuit

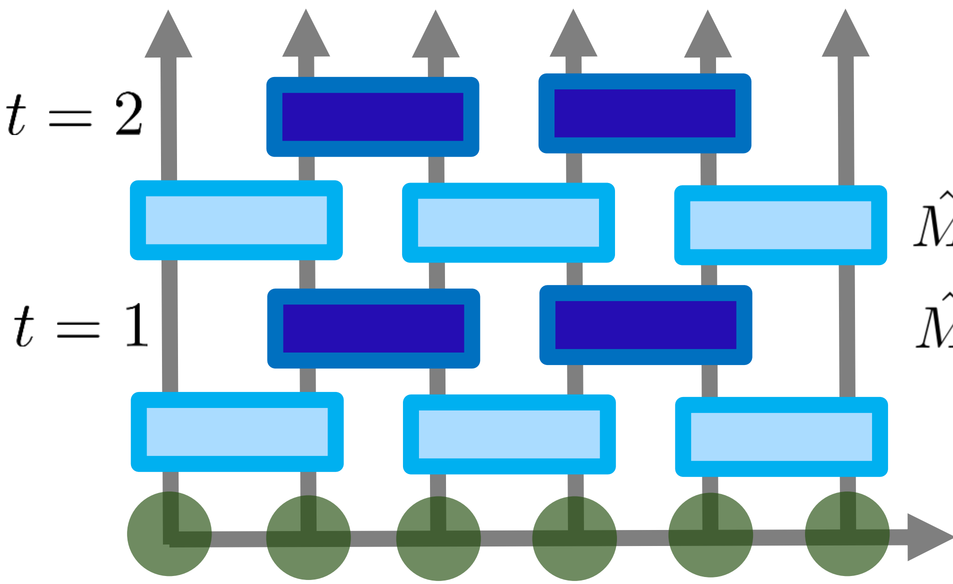
measurement operators on Majorana pairs:

$$\hat{\mathsf{M}}_{\omega_{t,y}}(\eta) \propto \hat{I} - i\omega_{t,y}\eta\hat{\gamma}_y\hat{\gamma}_{y+1} \qquad \omega_{t,y} = \pm \text{ : measurement outcomes}$$

Majorana operators:

$$\hat{\gamma}_{2x-1} = \hat{c}_x + \hat{c}_x^\dagger,\ i\hat{\gamma}_{2x} = \hat{c}_x - \hat{c}_x^\dagger,\quad \{\hat{c}_x, \hat{c}_{x'}^\dagger\} = \delta_{x,x'} \leftrightarrow \{\hat{\gamma}_y, \hat{\gamma}_{y'}\} = 2\delta_{yy'}$$

η : strength of measurement



$$\hat{M}^{\mathrm{o}}_{\omega_t}(\eta) = \prod_{y:\mathrm{odd}} \hat{\mathsf{M}}_{\omega_{t,y}}(\eta)$$

$$\hat{M}^{\mathrm{e}}_{\omega_t}(1-\eta) = \prod_{y:\mathrm{even}} \hat{\mathsf{M}}_{\omega_{t,y}}(1-\eta)$$

$$\omega_t = (\omega_{t,y=1}, \cdots, \omega_{t,y=2L})$$

H. Oshima, K. Mochizuki, R. Hamazaki, and Y. Fuji, Phys. Rev. Lett. **134**, 240401 (2025).

Single-particle Lyapunov analysis

transformation of Majorana operators:

$$\left[\hat{M}^{\dagger}_{\omega_t}(\eta)\right]\vec{\gamma}\left[\hat{M}^{\dagger}_{\omega_t}(\eta)\right]^{-1} = M_{\omega_t}(\eta)\vec{\gamma} \qquad \vec{\gamma} = \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \\ \cdots \\ \hat{\gamma}_{2L} \end{pmatrix}$$

single-particle time-evolution operator:

$$\Rightarrow \quad V_{\boldsymbol{\omega}_t}(\eta) = M^{\mathrm{e}}_{\omega_t}(1-\eta)M^{\mathrm{o}}_{\omega_t}(\eta)\cdots M^{\mathrm{e}}_{\omega_1}(1-\eta)M^{\mathrm{o}}_{\omega_1}(\eta)$$

single-particle effective "Hamiltonian":

$$\boxed{H_{\boldsymbol{\omega}_t}(\eta) = -\frac{i}{2t}\log\left[V_{\boldsymbol{\omega}_t}(\eta)V^{\dagger}_{\boldsymbol{\omega}_t}(\eta)\right]}$$

single-particle Hamiltonian ⇒ topological invariant: A. Y. Kitaev, Phys. Usp. **44**, 131 (2001).

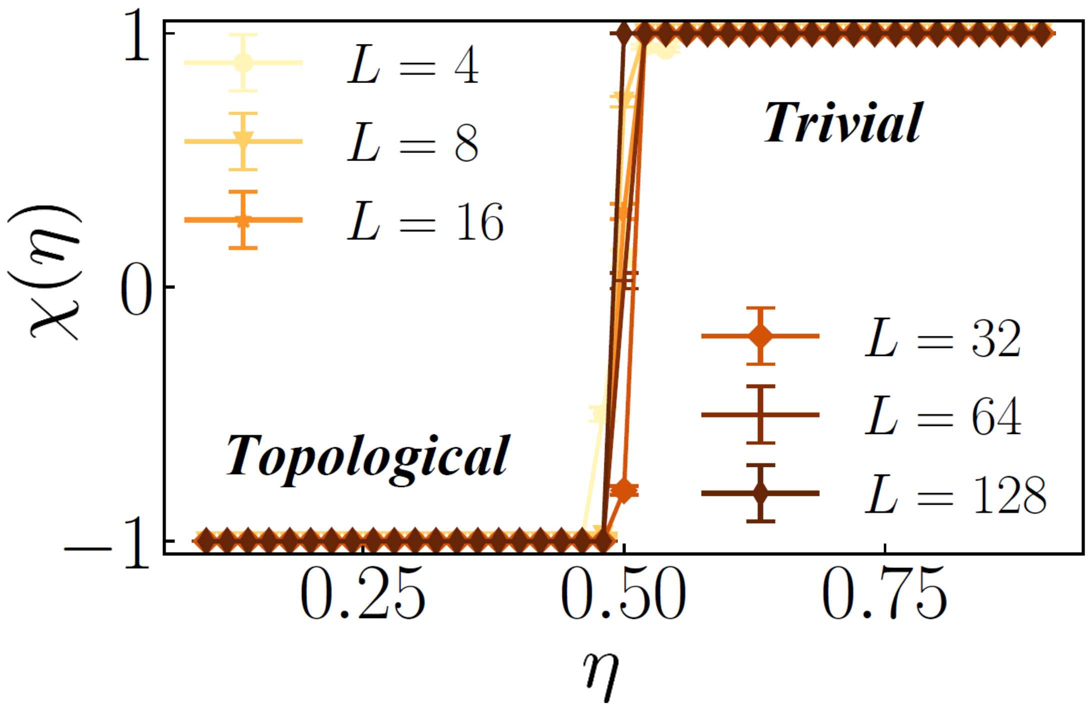
topological invariant: $\chi(\eta) = \lim_{t\to\infty}\mathrm{sign}\left(\mathrm{Pf}\left[H^{\mathrm{PBC}}_{\boldsymbol{\omega}_t}(\eta)\right]\mathrm{Pf}\left[H^{\mathrm{APBC}}_{\boldsymbol{\omega}_t}(\eta)\right]\right)$

H. Oshima, K. Mochizuki, R. Hamazaki, and Y. Fuji, Phys. Rev. Lett. **134**, 240401 (2025).

Topological invariant

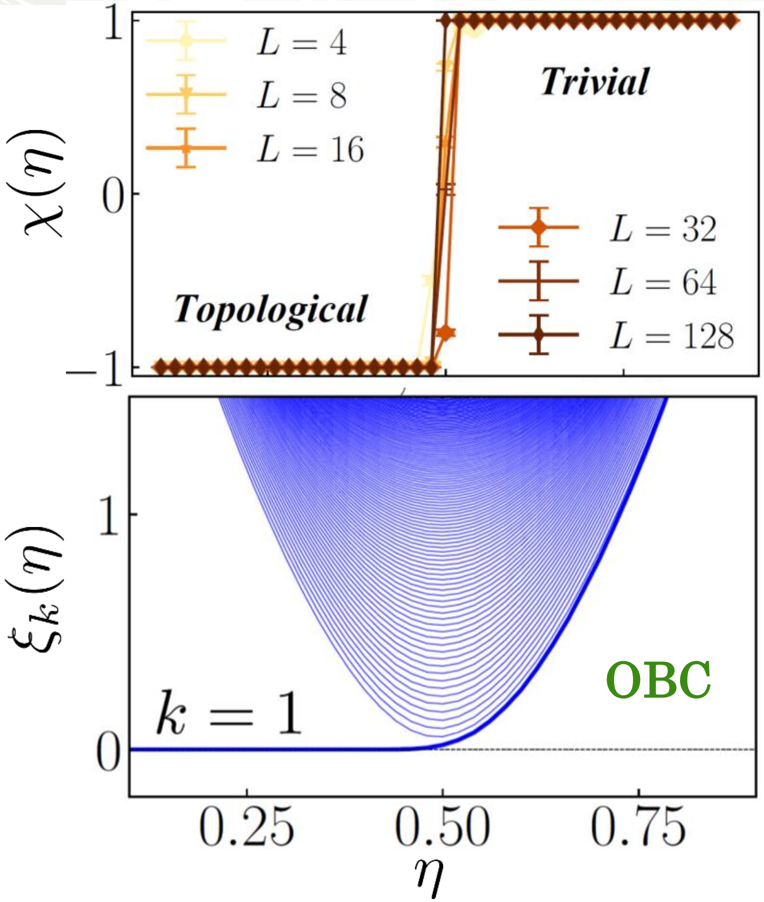
topological invariant: $\chi(\eta) = \lim_{t\to\infty} \mathrm{sign}\left(\mathrm{Pf}\left[H^{\mathrm{PBC}}_{\boldsymbol{\omega}_t}(\eta)\right]\mathrm{Pf}\left[H^{\mathrm{APBC}}_{\boldsymbol{\omega}_t}(\eta)\right]\right)$

*APBC : twisted measurement outcomes



H. Oshima, K. Mochizuki, R. Hamazaki, and Y. Fuji, Phys. Rev. Lett. **134**, 240401 (2025).

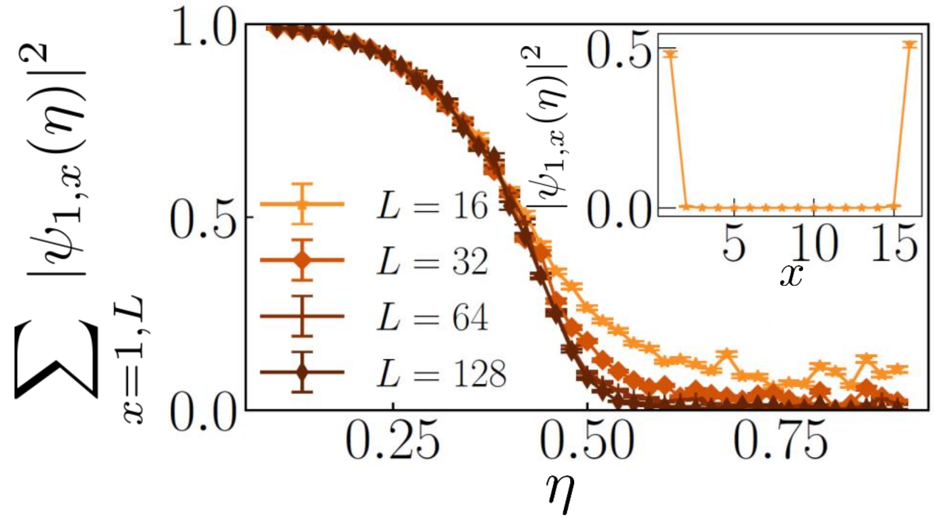
Bulk-edge correspondence



$\chi(\eta) = (-1)^{W(\eta)}$ PBC/APBC

bulk topological number $W(\eta)$

the number of edge modes under OBC



$$\lim_{t\to\infty} O^{\mathrm{T}}_{\boldsymbol{\omega}_t}(\eta) H_{\boldsymbol{\omega}_t}(\eta) O_{\boldsymbol{\omega}_t}(\eta) = \bigoplus_k \begin{bmatrix} 0 & +\xi_k(\eta) \\ -\xi_k(\eta) & 0 \end{bmatrix}$$

H. Oshima, K. Mochizuki, R. Hamazaki, and Y. Fuji, Phys. Rev. Lett. **134**, 240401 (2025).

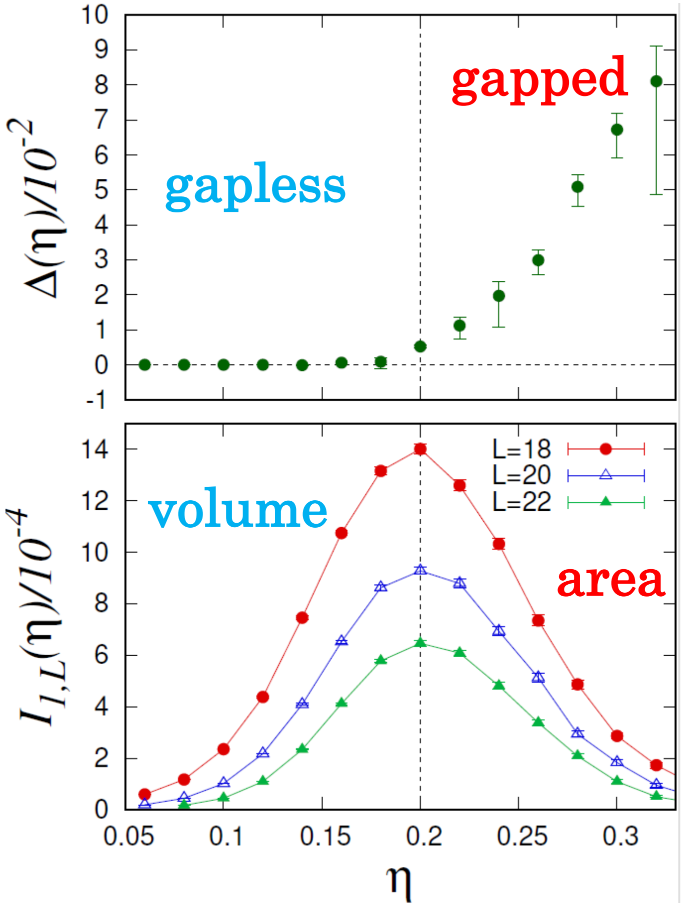
Outline

- Quantum phase transitions in isolated systems
- Measurement-induced transitions in monitored quantum systems
- Motivation and rough sketch of results
- Result-1: measurement-induced spectral transition in interacting spin systems
- Result-2: measurement-induced topological transition in non-interacting fermionic systems
- Summary

Summary

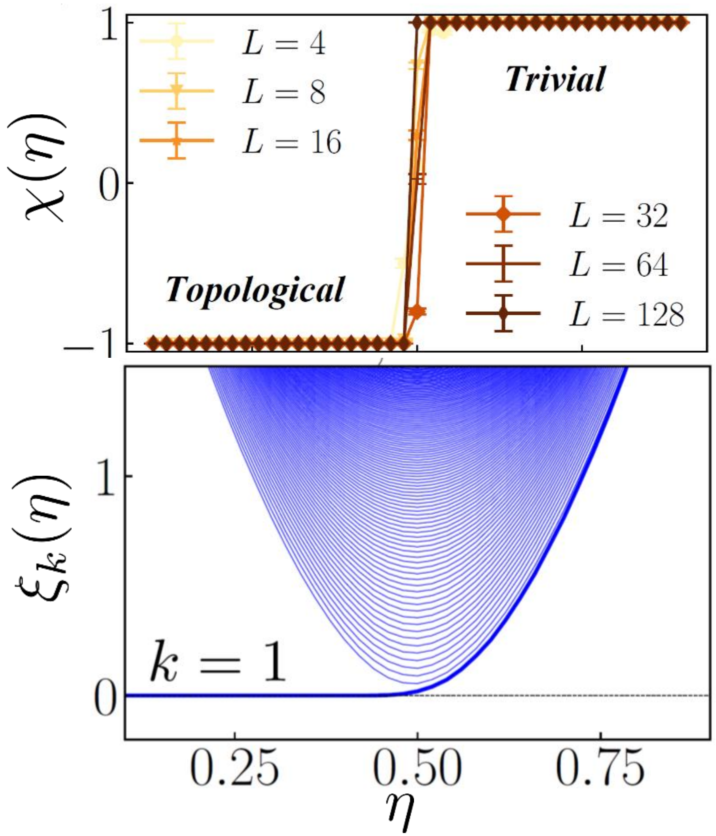
Phys. Rev. Lett. **134**, 010410 (2025).
Phys. Rev. Lett. **134**, 240401 (2025).

1. interacting spin system



2. non-interacting fermionic system

bulk-edge correspondence



spectrum and topology of the effective Hamiltonians
⇔ measurement-induced entanglement transitions

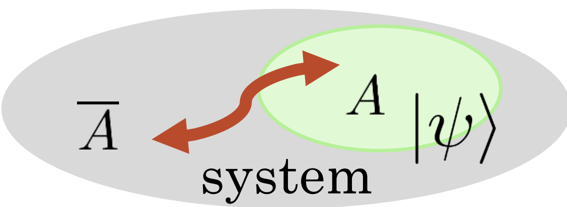
Entanglement of quantum states

$|\psi\rangle$: entangled? to what extent?

two spins: $|\psi\rangle = |\uparrow_1\downarrow_2\rangle$: not entangled $S_1(\psi) = 0$

$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle)$: entangled $S_1(\psi) = \log(2) \neq 0$

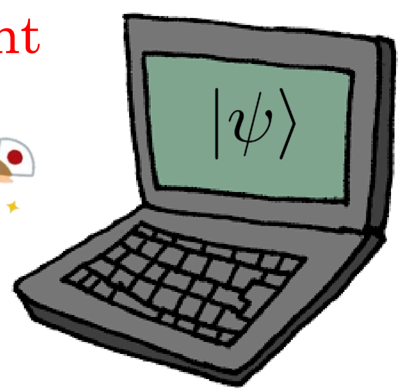
$$\boxed{\text{entanglement entropy: } S_A(\psi) = -\mathrm{tr}_A\left(\rho_A(\psi)\log\left[\rho_A(\psi)\right]\right)}$$



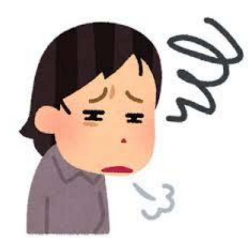
$$\rho_A(\psi) = \mathrm{tr}_{\bar{A}}\left(|\psi\rangle\langle\psi|\right)$$

large $S_A(\psi) \Rightarrow A$ and $\overline{A}$ are highly entangled

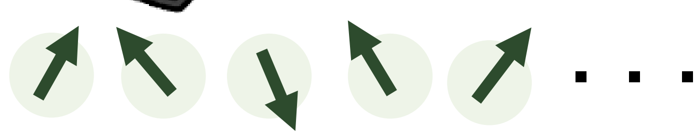
low entanglement



highly entangled



many spins on 1D chain:



Bulk-edge correspondence

gap closing ⟷ topological phase transition

flux-inserted
Kitaev chain

$\phi = 0$: PBC, $\phi = \pi$: APBC

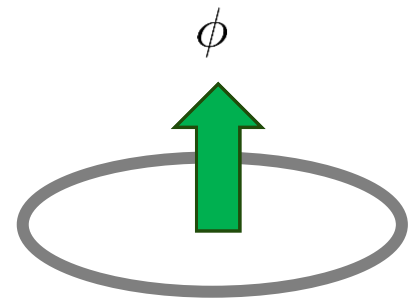
$$\hat{H}_\phi = \hat{H}_{\mathrm{bulk}} + e^{i\phi}\hat{c}_1^\dagger \hat{c}_L + e^{i\phi}\hat{c}_1\hat{c}_L + h.c.$$

$$\hat{H}_\phi \left|\Psi_i^\phi\right\rangle = \varepsilon_i^\phi \left|\Psi_i^\phi\right\rangle$$

parity conservation: $[(-1)^{\hat{N}}, \hat{H}_\phi] = 0$

Z_2 topological invariant based on the fermion parity

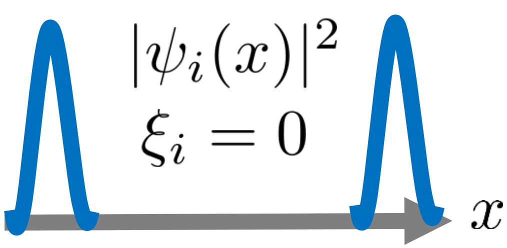
$$\chi = (-1)^W = \mathrm{sign}\left[\langle\Psi_1^0|(-1)^{\hat{N}}|\Psi_1^0\rangle\langle\Psi_1^\pi|(-1)^{\hat{N}}|\Psi_1^\pi\rangle\right]$$



topological phase $W = 1$	trivial phase $W = 0$

$\hat{N} = \sum_x \hat{c}_x^\dagger \hat{c}_x$

parameter μ



$W =$ **the number of Majorana edge modes under the open boundary condition**

Transition of purification timescale

mixed-state dynamics :

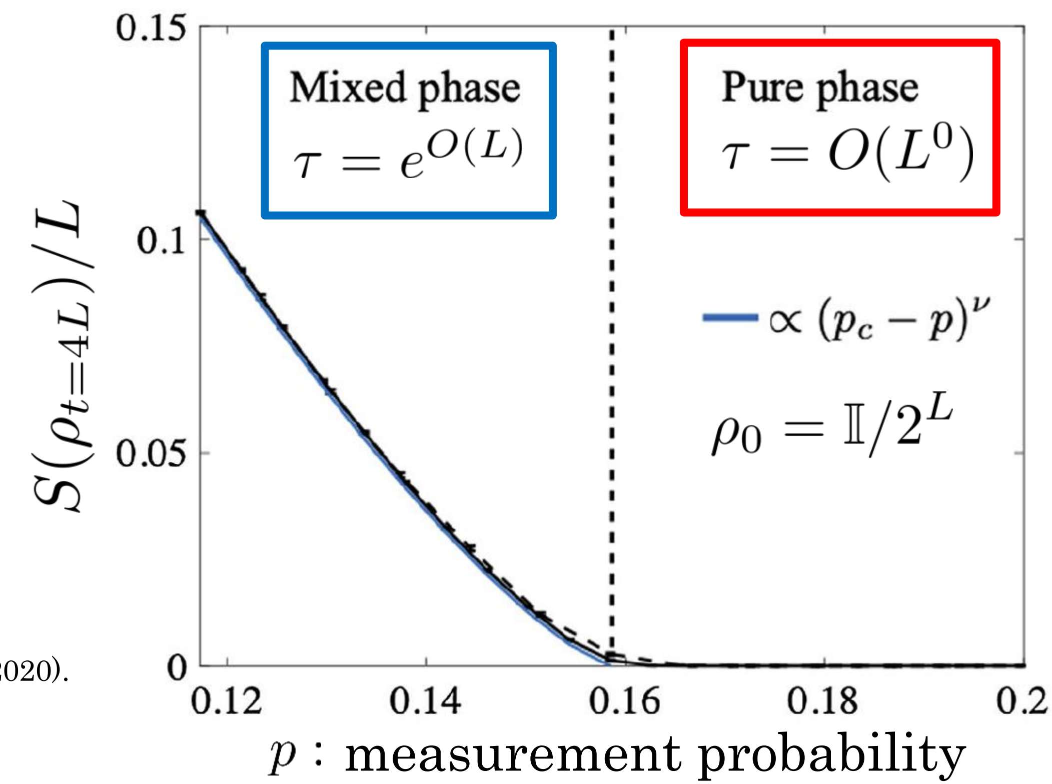
circuit $\rho_t \to \hat{\mathsf{U}}\rho_t\hat{\mathsf{U}}^\dagger$

measurement $\rho_t \to \dfrac{\hat{\mathsf{M}}_\omega\rho_t\hat{\mathsf{M}}_\omega^\dagger}{\mathrm{tr}\left(\hat{\mathsf{M}}_\omega\rho_t\hat{\mathsf{M}}_\omega^\dagger\right)}$

purification at τ : $S(\rho_\tau) \simeq 0$

$$S(\rho) = -\mathrm{tr}\left[\rho\log(\rho)\right]$$

purification transition in the mixed-state dynamics



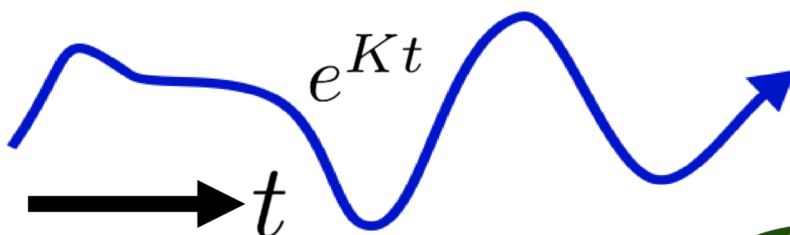
M. J. Gullans et.al., PRX **10**, 041020 (2020).

Spectrum in systems with no randomness

systems described by time-independent generators

$$\frac{d}{dt}\left|\psi(t)\right\rangle = -i\hat{H}\left|\psi(t)\right\rangle, \quad \frac{d}{dt}\hat{\rho}(t) = \mathcal{L}\hat{\rho}(t), \cdots$$

$\left|\psi(0)\right\rangle, \hat{\rho}(0)$



$\left|\psi(t)\right\rangle, \hat{\rho}(t)$

the spectrum of $K = -i\hat{H}, \mathcal{L}$

⬇

essential information of the system

$\det(\lambda - K) = \cdots$

Spectral gap in open quantum systems

Markovian open quantum systems

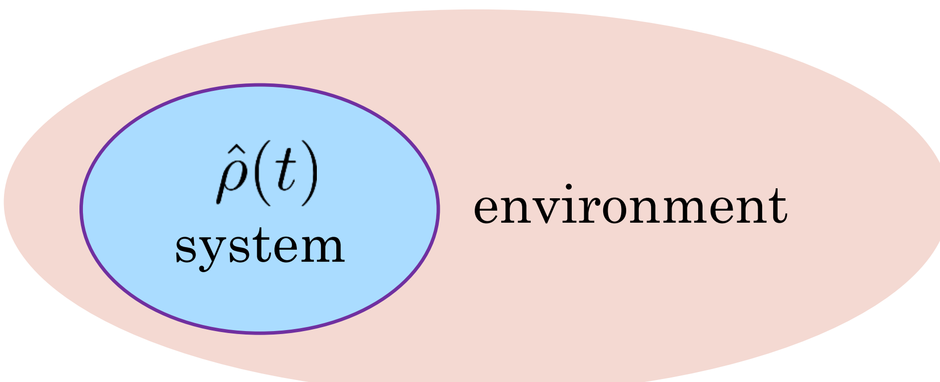
$$\frac{d}{dt}\hat{\rho}(t) = \mathcal{L}\hat{\rho}(t) \qquad \hat{\rho}(t) = e^{\mathcal{L}t}\hat{\rho}(0) = \sum_i c_i e^{\lambda_i t}\hat{\Psi}_i$$

the spectral gap $\Delta = |\mathrm{Re}(\lambda_2) - \mathrm{Re}(\lambda_1)|, \quad \mathrm{Re}(\lambda_j) \geq \mathrm{Re}(\lambda_{j+1})$

→ **the relaxation time to the stationary state** $1/\Delta$

$$\boxed{t \gg \frac{1}{\Delta} \to \hat{\rho}(t) \simeq \hat{\Psi}_1}$$

$$\mathcal{L}\hat{\Psi}_i = \lambda_i \hat{\Psi}_i$$

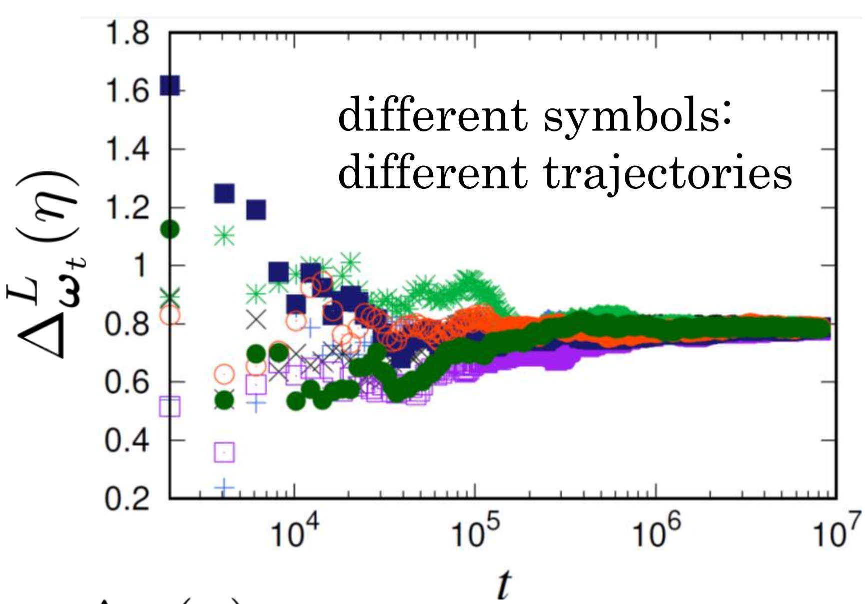


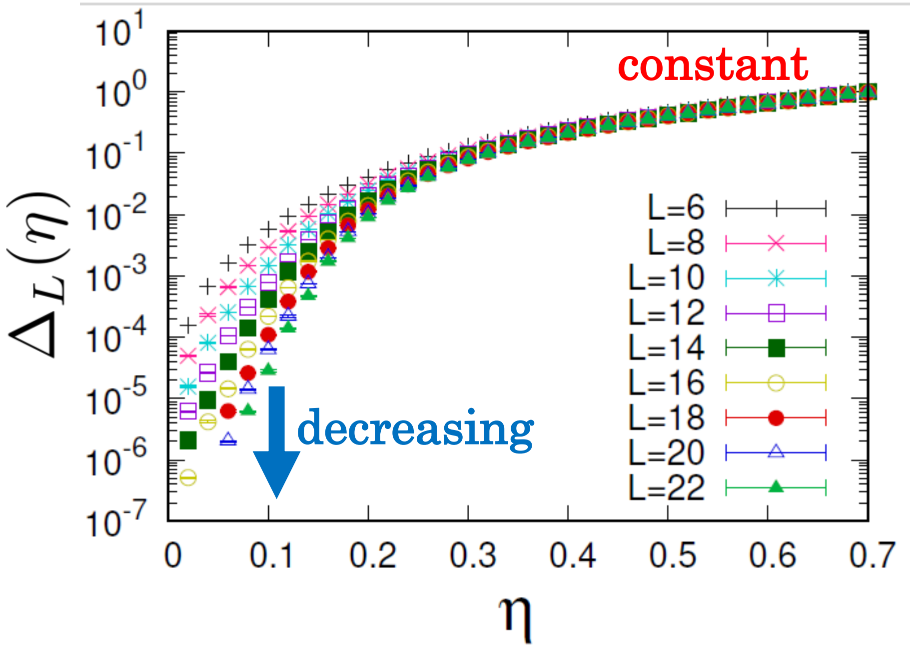
Spectral gap

Ken Mochizuki
Ryusuke Hamazaki
PRL **134**, 010410 (2025).

$$\hat{H}_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle = \varepsilon^{i,L}_{\boldsymbol{\omega}_t}(\eta)\,|\Psi^i_{\boldsymbol{\omega}_t}(\eta)\rangle\,, \quad \varepsilon^{i,L}_{\boldsymbol{\omega}_t}(\eta) \le \varepsilon^{i+1,L}_{\boldsymbol{\omega}_t}(\eta)$$

spectral gap: $\Delta_L(\eta) = \lim_{t\to\infty} \Delta^L_{\boldsymbol{\omega}_t}(\eta),\ \Delta^L_{\boldsymbol{\omega}_t}(\eta) = \varepsilon^{2,L}_{\boldsymbol{\omega}_t}(\eta) - \varepsilon^{1,L}_{\boldsymbol{\omega}_t}(\eta)$





$\Delta_L(\eta)$: independent of trajectories

global quantity!

η : measurement strength

L : system size

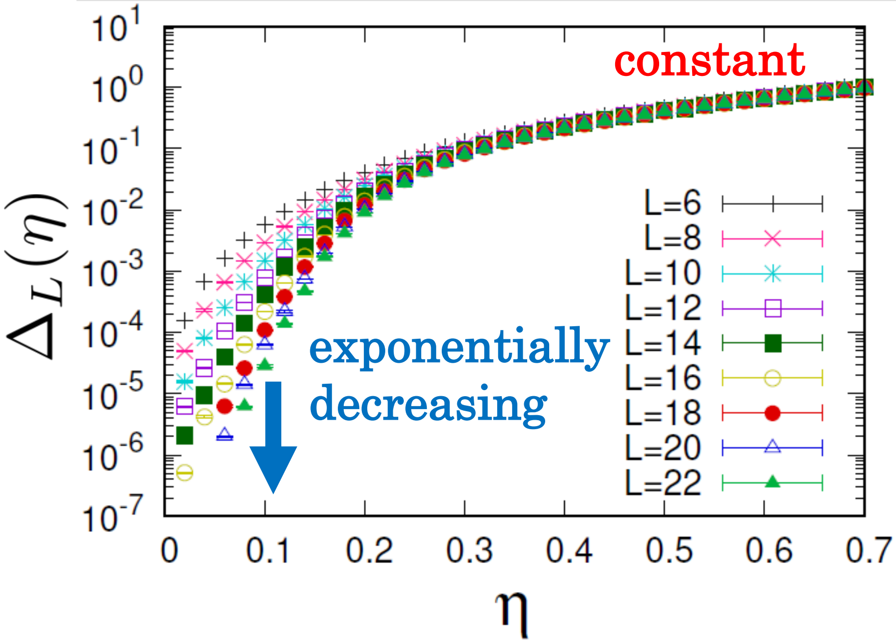
small η : $\lim_{L\to\infty} \Delta_L(\eta) = 0$

large η : $\lim_{L\to\infty} \Delta_L(\eta) \neq 0$

Spectral transition

Ken Mochizuki
Ryusuke Hamazaki
PRL **134**, 010410 (2025).

spectral gap: $\Delta_L(\eta) = \lim_{t\to\infty} \Delta^L_{\boldsymbol{\omega}_t}(\eta),\ \Delta^L_{\boldsymbol{\omega}_t}(\eta) = \varepsilon^{2,L}_{\boldsymbol{\omega}_t}(\eta) - \varepsilon^{1,L}_{\boldsymbol{\omega}_t}(\eta)$

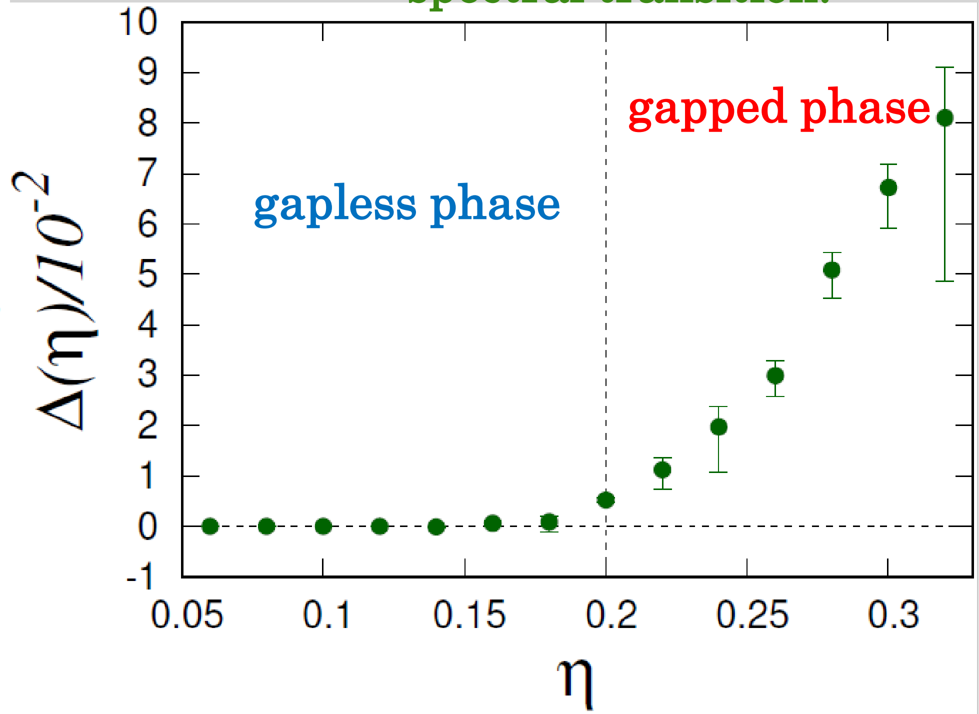


fitting:

$$\Delta^{\rm fit}_L(\eta) = \Delta(\eta) + \alpha(\eta)[\beta(\eta)]^{-L}$$

$$\Delta(\eta) = \lim_{L\to\infty} \Delta^{\rm fit}_L(\eta)$$

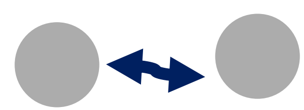
spectral transition!



Comparison of transitions

	Monitored dynamics where Hamiltonians can be non-local		Well-known equilibrium systems described by local Hamiltonians
Gapped phase	$S_{L/2} = O(L^0)$ $\Delta_L = O(L^0)$	same	$S_{L/2} = O(L^0)$ $\Delta_L = O(L^0)$
Gapless phase	$S_{L/2} = O(L)$ $\Delta_L = \exp\left[-O(L)\right]$	distinct	$S_{L/2} = O[\log(L)]$ $\Delta_L = O[1/\mathrm{poly}(L)]$

long-range interaction ⇒ volume law?



*1D systems
e.g. XXZ model

T. Udagawa and H. Katsura,
J. Phys. A: Math. Theor. **50** (2017) 405002.

B. Zeng, X. Chen, D.-L. Zhou, and X.-G. Wen,
Quantum information meets quantum matter (Springer, 2019).

Ken Mochizuki, Ryusuke Hamazaki, Physical Review Letters **134**, 010410 (2025).

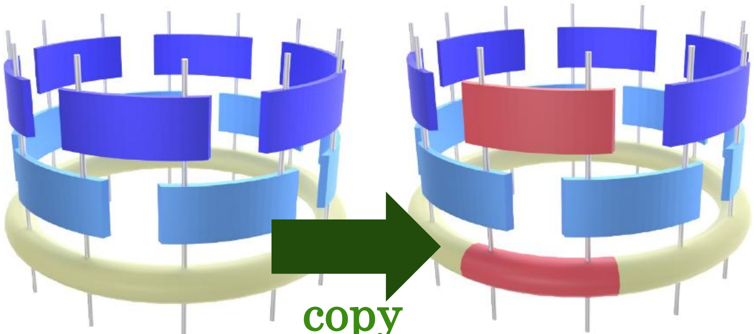
Topological invariant

ground-state fermion parity:

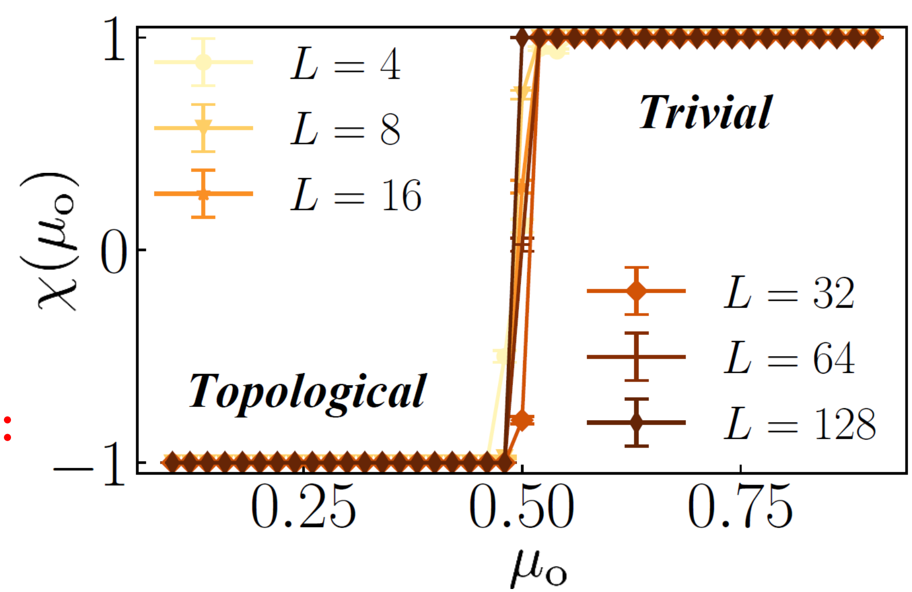
$$\lim_{t\to\infty} \langle \Psi_{1,\boldsymbol{\omega}_t}(\mu_{\mathrm{o}})| (-1)^{\hat{N}} |\Psi_{1,\boldsymbol{\omega}_t}(\mu_{\mathrm{o}})\rangle = \lim_{t\to\infty} \mathrm{sign}\left(\mathrm{Pf}\left[H_{\boldsymbol{\omega}_t}(\mu_{\mathrm{o}})\right]\right)$$

topological invariant: $\chi(\mu_{\mathrm{o}}) = \lim_{t\to\infty} \mathrm{sign}\left(\mathrm{Pf}\left[H^{\mathrm{PBC}}_{\boldsymbol{\omega}_t}(\mu_{\mathrm{o}})\right] \mathrm{Pf}\left[H^{\mathrm{APBC}}_{\boldsymbol{\omega}_t}(\mu_{\mathrm{o}})\right]\right)$

PBC



APBC : twisted measurement outcomes



periodic boundary condition (PBC):

$$\hat{\mathrm{M}}_{\omega_{t,2L}}(\mu_{\mathrm{e}}) \propto \hat{I} - i\omega_{t,2L}\mu_{\mathrm{e}}\hat{\gamma}_{2L}\hat{\gamma}_1$$

anti-periodic boundary condition (APBC):

$$\hat{\mathrm{M}}_{\omega_{t,2L}}(\mu_{\mathrm{e}}) \propto \hat{I} + i\omega_{t,2L}\mu_{\mathrm{e}}\hat{\gamma}_{2L}\hat{\gamma}_1$$

other outcomes : same

H. Oshima, K. Mochizuki, R. Hamazaki, and Y. Fuji, Phys. Rev. Lett. **134**, 240401 (2025).